Special Distributions

1 Binomial Distribution (James Bernoulli)

Consider a random experiment, which is so defined that it has only two possible outcomes which we call *success* and *failure*. Let p be the probability of a success and q = 1-p be the probability of failure. Let the experiment be repeated n times. Also assume that the trails are independent and the probability of success p remains unaltered by trail to trail. Then obviously the number of success in n trails is a random variable. Let X denote the number of successes in n independent repetitions of this experiment and let $f(x)$ be the p.d.f. of X.

If the x repetitions in which success occurs are specified, the probability of success in these x repetitions and failures in the remaining $n - x$ repetitions is $p^x q^{n-x}$ by the multiplication theorem the repetitions being independent. The x trials in which success occurs may be specified in $\binom{n}{x}$ mutually exclusive ways. So the event $X = x$ can occur in $\binom{n}{x}$ mutually exclusive ways and probability of each is $p^x q^{n-x}$. So by addition theorem, the required probability is $\binom{n}{x} p^x q^{n-x}$.

i.e.

$$f(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, \ldots, n, 0 \le p \le 1.$$

This distribution is called binomial distribution.

Definition:- A discrete random variable X is said to follow a binomial distribution with parameters n and p if its probability density function is given by

$$f(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, \ldots, n, 0 \le p \le 1, p+q=1.$$

Properties

1. Mean = E(X) = np and variance = V(X) = npq.
2. If $(n+1)p$ is integer the binomial distribution has two mode one at $(n+1)p$ -1 and the other at $(n+1)p$. If $(n+1)p$ is not an integer then the integer part of $(n+1)p$ is the mode.
3. If $p = q = ½$ the distribution is symmetrical and when $p \neq q$ the distribution is a skewed distribution.
4. If X_i (i=1, 2, …, k) are independent binomial random variate with parameters (n_i, p). Then $S_k = \sum_{i=1}^{k} X_i$ has a binomial distribution with parameters ($\sum_{i=1}^{k} n_i$, p). (This property is called additive or reproductive property).
5. If n is very large and if neither p and q is too close to zero binomial distribution may be approximated by normal distribution.

2 Normal Distribution

A continuous random variable X with p.d.f.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\}, \ -\infty<x<\infty$$

is said to follow normal distribution with parameters (μ, σ).

Properties

1. The normal curve is bell shaped, unimodal, symmetric about the mean and approaches to the x axis at $\pm\infty$.
2. The mean, mode and median coincide at $x = \mu$ and σ is the standard deviation.
3. The odd central moments are zero.
4. The curve is mesokurtic
5. It has points of inflexion at $x = \mu \pm \sigma$.
6. The mean deviation about the mean is $\frac{4}{5}\sigma$
7. The quartile deviation is $\frac{2}{3}\sigma$
8. The interval $(\mu - 2\sigma, \mu + 2\sigma.)$ contains 95% of the area and $(\mu - 3\sigma, \mu + 3\sigma.)$ contains 99% of the area under the curve.
9. If X has normal distribution with parameter (μ, σ) and a and b are any two real numbers then the random variable $Y = aX + b$ is also a normal distribution with parameters ($a\mu + b, a\sigma$).
10. If X and Y are independent normal vairate with mean and SD (μ_1, σ_1) and (μ_2, σ_2) and a and b are any two real numbers then the random variable $Z = aX + bY$ is also a normal distribution with mean and SD ($a\mu_1 + b\mu_2$, $\sqrt{a^2\sigma_1^2 + b^2\sigma_2^2}$).

Standard Normal Distribution

A normal distribution with mean zero and variance one is called a standard normal distribution. That is the pdf of the standard normal distribution is

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{\frac{-x^2}{2}\right\}, \ -\infty<x<\infty$$

Note

If X is a normal (μ, σ) then $Z = \dfrac{x-\mu}{\sigma}$ has standard normal distribution.