Skewness and Kurtosis

Skewness

In everyday language, the terms "skewed" and "askew" are used to refer to something that is out of line or distorted on one side. When referring to the shape of frequency or probability distributions, "skewness" refers to asymmetry of the distribution. A distribution with an asymmetric tail extending out to the right is referred to as "positively skewed" or "skewed to the right," while a distribution with an asymmetric tail extending out to the left is referred to as "negatively skewed" or "skewed to the left." Skewness can range from minus infinity to positive infinity.

Measure of Skewness

A measure of Skewness should indicate by its sign whether the skewness is positive or negative and by its magnitude the degree of skewness. The following are some measures of skewness

1. Karl Pearson Measure of Skewness

Karl Pearson (1895) first suggested measuring skewness by standardizing the difference between the mean and the mode and is denoted by J, that is,.

$$J = \frac{Mean - Mode}{SD}$$

Population modes are not well estimated from sample modes, but one can estimate the difference between the mean and the mode as being three times the difference between the mean and the median (Stuart & Ord, 1994), leading to the following estimate of skewness: .

$$J = \frac{3(Mean - Median)}{SD}$$

This statistic ranges from -3 to +3. Absolute values above 0.2 indicate great skewness (Hildebrand, 1986).

2. Quartile Measure of Skewness

Quartile Measure of Skewness is based on the three quartiles and is defined as

$$Sk_Q = Q_3 + Q_1 - 2M$$

or

$$Sk_Q = Q_3 + Q_1 - 2Q_2$$

3. Bowely Measure of Skewness

Bowely Measure of Skewness or Bowely coefficient measure of Skewness is defined as

$$Sk_Q = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$$

Or

$$Sk_Q = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

This statistic ranges from -1 to +1.

4. Karl Pearson Measure of Skewness

Karl Pearson moment measure of skewness or Karl Pearson coefficient measure of skewness or beta and gamma measure of skewness is defined as

$$\beta_1 = \frac{m_3^2}{m_2^3} \text{ and } \gamma_1 = \sqrt{\beta_1}.$$

The sign of the third central moment m_3 indicates whether the skewness is positive or negative. Also β_1 is scale and location invariant. For a symmetric distribution β_1=0

Kurtosis

Pearson (1905) introduced kurtosis as a measure of how flat the top of a symmetric distribution is when compared to a normal distribution of the same variance. Thus kurtosis measurers the flatness or peakedness of a distribution curve from the normal curve. In other words kurtosis measurers the concentration of the observation in the neighbourhood of the mode.

If the observation in the neighbourhood of the mode may be much less than the normal curve, then the curve will be much flatter than the normal curve This less flat-topped distributions as is called "platykurtic" . If the concentration is more in the neighbourhood of the mode of the normal curve the distributions is known as "leptokurtic" and equally flat-topped distributions or normal distribution are called "mesokurtic". Kurtosis is actually more influenced by scores in the tails of the distribution than scores in the center of a distribution. Accordingly, it is often appropriate to describe a leptokurtic distribution as "fat in the tails" and a platykurtic distribution as "thin in the tails."

Measure of Kurtosis

There are two measures of kurtosis namely Quartile measure of kurtosis and Karl Pearson moment measure of kurtosis

1. Quartile measure of kurtosis

Quartile measure of kurtosis is defined as $k = \frac{\frac{1}{2}(Q_3 - Q_1)}{Q_{0.9} - Q_{0.1}}$.

1. For a platykurtic distribution k tends to Zero
2. For a leptokurtic distribution k tends to 0.5
3. For a mesokurtic distribution k tends to 0.25.

2. Karl Pearson Measure of Kurtosis

Karl Pearson moment measure of kurtosis or Karl Pearson coefficient measure of kurtosis or beta and gamma measure of kurtosis is defined as

$$\beta_2 = \frac{m_4}{m_2^2} \text{ and } \gamma_2 = \beta_2 - 3.$$

1. For a platykurtic distribution β_2 will be less than 3 and γ_2 is negative
2. For a leptokurtic distribution β_2 will be more than 3 and γ_2 is positive
3. For a mesokurtic distribution $\beta_2 = 3$ and γ_2=0.