Bayes Estimation of Reliability under Stress-Strength Model when Stress is Censored at Strength

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Abstract

In the context of reliability the stress-strength model describes the life a component which has a random strength X and is subject to a stress Y. The component fails at the instant the stress applied to it exceeds the strength and the component will work satisfactory when ever X>Y. Thus R=P[Y<X] is a measure of system reliability. In the present paper we derive the Bayes estimate of R when stress Y is censored at strength X for the Marshall-Olkin bivariate exponential distribution and the bivariate Pareto distribution due to Muliere and Scarsini (1987).

1. Introduction

The problem of estimation $R = \mathrm{P}(Y<X)$ was first dealt only with the non-parametric setup in the late fifties and early sixties. The problem of point and interval estimation of $\mathrm{P}(Y<X)$ was considered by Birnbaum (1956), Birnbaum and McCarty (1958), Govindarajulu (1967,1968), Owen et. al. (1964), Sen (1960, 1967), Van Dantzing (1951) and Zaremba (1965). In the seventies the problem of estimation in the parametric setup was dealt with mainly in the normal, exponential and the Pareto distributions. Among them Kelly et. al. (1976), Tong (1974), Church and Harris (1970), Downton (1973), Woodward and Kelley (1977), Beg and Singh (1979), Tong (1977) etc considered the problem in the classical frame work while Enis and Gisser (1971), Ferguson (1973), Hollander and Korwar(1976), Bhattacharyya and Johnson (1974) look the problem under the Bayesian frame work.

During eighties the estimation of the P(*Y*<*X*) get momentum and the estimators was obtained for the majority of the distribution when *X* and *Y* are independent, Awad and Gharraf (1986), Beg (1980a, b, c), Constantine et. al. (1986), Ismail et. al. (1986), Iwase (1987), Reiser and Guttman (1986), Voinov (1984)). The theory of estimation was extended to the case of mixture distributions (Akman et al. (1999)), outlier cases (Jeevanand and Nair (1994), Jeevanand (1998a)), elliptical distribution (Pensky (2002)), Skew-normal (Azzalini and Chiogna (2002)) during late nineties and in 2000.

The estimation problem when *X* and *Y* have dependent case or *X* and *Y* have a joint bivariate distribution was considered by Abu-Salih and Shamseldin (1988), Awad et al. (1981), Basu (1981), Jana (1994), Jana and Roy (1994), Jeevanand (1997, 1998b), and Hangal (1995, 1997a, b), Cramer (2001). Johnson (1988) in his review paper "Stress-Strength Model for Reliability" in the Hand book of Statistics, Vol.7 gives a very good survey of the literature up to 1998 on the estimation problem and Kotz et al. (2003) in their book "The Stress-Strength Model and its Generalizations" gives almost a very complete survey and bibliography on the subject.

The stress-strength relationship is now a days studied in many branches of sciences and social sciences such as psychology, medicine, pedagogy, pharmaceutics and engineering. The book by S. Kotz, Y.Lumelskii and M.Pensky (2003), "The Stress-Strength Model and its Generalizations" and the review paper by R.A. Johnson (1988), "Stress-Strength Model for Reliability" in the Hand book of Statistics, Vol.7 has cited many applications of the model in engineering, reliability, quality control, medicine and psychology.

Hangal (1997a) consider the problem of estimation of R = P(*Y*<*X*) for the Marshall-Olkin bivariate exponential distribution when the stress is censored at the strength. But It appears that the corresponding problem within the Bayesian frame work has not been discussed in literature and accordingly in this article we propose the Bayes estimate of *R* when (*X,Y*) has Marshall-Olkin bivariate exponential distribution and the stress is censored at strength. The estimates are obtained under a class of loss functions. Lindley and Singpurwalla (1986) pointed out that the distribution of life lengths measured in a laboratory environment as exponential distribution provided that, when they work in a different environment which may be harsher, the same or gentler than the original, the resulting density of life lengths has a Pareto distribution. To accomadate these situation in the present paper wefid the estimate of the R when stress is censored at strength when (*X,Y*) has bivariate Pareto distribution due to Muliere and Scarsini (1987). The problem of estimation of R based on complete sample for the bivariate Pareto distribution was discussed in the literature by Hangal (1996, 1997b) and Jeevanand (1997).

The problem investigated here has relevance in the context of analysing the reliability of component whose strength is represented by the random variable *X* which is subjected to a stress *Y*. The component fails at the instant the stress applied to it exceeds the strength and it will work properly if there is no other cause of failure when ever$Y<X$. Thus R = P($Y<X$) is an important measure of component reliability. Also there are many situation where it is not possible to take the complete sample due to lack of time or minimization of the experiment cost or experiments leads to the distraction of a component. Some of the practical situation of the model are as follows (c.f. Hangal (1997b)) .

A rubber has certain strength which is elastic in nature. If the rubber is stretched more than its strength, it leads to the destruction of rubber and stress cannot be recorded. Similar examples are spring and balloon which can be stretched to a certain level; beyond that level, stretching leads to destruction of the component.

Now suppose that $\underline{x} = (x_1, \ldots, x_n)$ is a random sample from a distribution with pdf $f(x,\lambda)$, where $\lambda \in R$ is an unknown parameter.

Let the parameter λ has a prior probability $g(\lambda)$. After observing the data x, this prior density can be updated to the posterior density using the Bayes theorem

$$f(\lambda|\underline{x}) \propto \ell(\underline{x}|\lambda)\ g(\lambda) \qquad (1.1)$$

where $\ell(\underline{x}|\lambda)$ is the likelihood function of the observations x. For the purpose of estimation we consider the following three loss function

(i) Asymmetric linear Loss function defined by

$$L(\delta,\lambda) = \begin{cases} a(\delta-\lambda) & \text{if } \lambda \le \delta \\ b(\lambda-\delta) & \text{if } \lambda > \delta \end{cases} \qquad (1.2)$$

where a, b>0. This function is asymmetric for $a \neq b$. Under this loss function the Bayes estimate of λ is the $b/(a+b)$-quintile of the posterior distribution of λ. When a=b, the loss function reduces to the usual absolute error loss function and the estimate is the median of the posterior density of λ.

(ii) Symmetric loss function

$$L(\delta,\lambda) = (\delta-\lambda)^{2k},\ k = 1, 2, \ldots \qquad (1.3)$$

in this case the Bayse estimate is obtained as the solution of the equation

$$\sum_{j=0}^{2k} \binom{2k}{j} (-1)^j\, j\delta^{j-1} E(\lambda^{2k-j}|\underline{x}) = 0.$$

When k =1 the loss function reduces to usual squared error loss function and the Bayes estimate is the posterior expectation of λ.

(iii) The Linex loss function defined by

$$L(\delta,\lambda) = b[a(\delta-\lambda) + e^{-a(\delta-\lambda)} - 1] \qquad (1.4)$$

where a, b>0. Under this loss function the Bayes estimate is

$$\delta = \frac{1}{a}\ln\left(\int_{-\infty}^{\infty} e^{a\lambda} f(\lambda|\underline{x})\,d\lambda\right).$$

The remaining part of the paper is organized as follows. In Section 2 we consider the estimation problem when the underlying distribution is bivariat exponential due to Marshal and Olkin (1967). In section 3 we obtain the bayes estimate of bivariate Pareto distribution due to Muliere and Scarsini (1987). Final in the last section we asses the performance of the estimates obtained so far empirically.

2. Estimation for Marshal Olkin Bivariate Exponential Model

Now consider a component which has a random strength *X* subjected to a stress Y. Where *X* and *Y* have initially independent exponential distribution. But as the equipment goes on working, the strength of the component slowly depends on the stress. Thus after some time the strength X will have a well-established dependence on Y and accordingly we assume that (*X,Y*) have the bivariate Marshal-Olkin (1967) exponential distribution, with joint survival function

$$\overline{F}(x,y) = P[X>x, Y>y] = \exp\{-\lambda_1 x - \lambda_2 y - \lambda_3 \max(x,y)\} \quad (2.1)$$

where $x \geq 0, y \geq 0$. Then

$$R = P(Y<X) = \frac{\lambda_2}{\lambda}, \quad (2.2)$$

where $\lambda = \lambda_1 + \lambda_2 + \lambda_3$.

Let (x_i, y_i), i=1, 2, ..., n be a random sample of size n from (2.1) and that Y is censored X. Define $Z = \min(X,Y)$. Thus we have

$$(x_i, y_i) = (x_i, y_i) \quad \text{if} \quad x_i > y_i$$
$$= (x_i, x_i) \quad \text{if} \quad x_i \leq y_i.$$

Then the likelihood of the sample is given by Hangal (1997a) as

$$\ell(x,y \mid \Lambda) = \gamma^n \lambda_2^{n_2} \exp\{-\gamma t_x - \lambda_2 t_z\} \quad (2.3)$$

where n_2 is the number of observation with $x_i > y_i$, $\gamma = \lambda_1 + \lambda_3$, $t_x = \sum_{i=1}^{n} x_i$ and $t_z = \sum_{i=1}^{n} z_i$.

A suitable prior for γ and λ_2 is

$$g(\gamma, \lambda_2) = C_1 \ \gamma^{r-1} \lambda_2^{m-1} \exp\left\{-\gamma t_x^{'} - \lambda_2 t_z^{'}\right\} \qquad (2.4)$$

Here after C with various suffix denote the normalizing constant. Combining (2.3) and (2.4) we obtain the posterior density of (γ, λ_2) as

$$f(\gamma, \lambda_2 \,|\, \underline{x}, \underline{y}) = C_2 \ \gamma^{N-1} \lambda_2^{M-1} \exp\left\{-\gamma T_x - \lambda_2 T_z\right\} \qquad (2.5)$$

where$N = n+r$, $M = n_2+m$, $T_x = t_x + t_x^{'}$ and $T_z = t_z + t_z^{'}$.

Now using the transformation $R = \dfrac{\lambda_2}{\lambda_2 + \gamma}$ and $W = \gamma$ and integrating out W we obtain the posterior density of R as

$$f(R|\underline{x}, \underline{y}) = [C_3(0)]^{-1} R^{M-1} (1-R)^{N-1} (1-TR)^{-(N+M)}, \quad 0 \le R \le 1, \qquad (2.6)$$

where $T = 1 + \dfrac{T_z}{T_x}$,

$$C_3(d) = B(N, M+d) \ {}_2F_1(N+M, M+d, M+N+d; T) \qquad (2.7)$$

and

$${}_2\mathbf{F}_1(a,b,c{:}x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{j=0}^{\infty} \frac{\Gamma(a+j)\Gamma(b+j)}{\Gamma(c+j)\Gamma(j+1)} x^j$$

is the Gauss hypergeometric function.

The Bayes estimate R_1 under the asymmetric linear loss function (1.2) is obtained as the solution of the equation

$$[C_3(0)] \int_0^{R_1} R^{M-1}(1-R)^{N-1}(1-TR)^{-(N+M)} dR = \frac{b}{a+b}. \qquad (2.8)$$

For the Bayes estimate R_2 under the symmetric loss (1.3) we solve the equation

$$\sum_{j=0}^{2k} \binom{2k}{j} (-1)^j j \delta^{j-1} \frac{C_3(2k-j)}{C_3(0)} = 0. \qquad (2.9)$$

When $k = 1$ the Bayes estimate R_2 reduces to

$$R_2 = C_3(1) \,/\, C_3(0). \qquad (2.10)$$

Finally the Bayes estimate under Linex loss function (1.4) is given by

$$R_3 = \frac{1}{a}\ln\left([C_3(0)]\int_0^1 e^{aR}R^{M-1}(1-R)^{N-1}(1-TR)^{-(N+M)}dR\right) \qquad (2.11)$$

which can be solved numerically for R_3.

3. Estimation for Bivariate Pareto Model

In this section we discuss the problem of estimation of $R$when X and Y has bivariate Pareto distribution of Muliere and Scarsini (1987) which arises in the following context.

Consider a two-component system that is subject to shocks arriving from sources 0, 1 and 2 independently according to Poisson processes with respective intensities θ_0, θ_1 and θ_2. Let X and Y be the life of the first and second component with a minimum guarantee period β of failure free operation (i.e. $\min(X,Y)>\beta$). Define $X=\min(U_0,U_1)$ and $Y=\min(U_0,U_2)$ where U_j, j= 0, 1, 2, denotes the waiting time for the first shock from source j with probability density function

$$f(u_j) = \theta_j \exp\{-\theta_j(u_j-\beta)\} \qquad (3.1)$$

then (X,Y) have the Marshal-Olkin (1967) bivariate exponential distribution. Now the effect of change in operating conditions in which the intensity of forces applied to the system are random is accommodated by assuming that θ_j, 0, 1, 2, is gamma(λ_j,β) with pdf

$$f(\theta_j) = \frac{\beta^{\lambda_j}}{\Gamma(\lambda_j)}\theta_j^{\lambda_j-1}\exp\{-\beta\theta_j\}. \qquad (3.2)$$

Averaging over θ_j we obtain the joint survival function of (X,Y) as

$$\overline{F}(x,y) = P[X>x,Y>y] = \left(\frac{x}{\beta}\right)^{-\lambda_1}\left(\frac{y}{\beta}\right)^{-\lambda_2}\max\left(\frac{x}{\beta},\frac{y}{\beta}\right)^{-\lambda_3}. \qquad (3.3)$$

The distribution given above has wide application in medical and pharmaceutical science and engineering. Because of the real life application of the model, the estimation of R=P($Y<X$) and related inference is important and requires investigation. The classical estimation of R when β=1 for uncensored data was discussed in Hangal (1997a). In the section we obtain the Bayes estimate of R

when Y is censored at X in the case of known β. The assumption that β is known is realistic as it is normally the warranty period of the equipment. The expression for R when (X,Y) has distribution (3.3) is given by Jeevanand (1997) as

$$R = P(Y<X) = \lambda_2/\lambda \tag{3.4}$$

where $\lambda = \lambda_1 + \lambda_2 + \lambda_3$.

Let (x_i, y_i), i=1, 2, …, n be a random sample of size n from (3.3) and that Y is censored X. Then with the assumption as in the section 2 the likelihood can be written as

$$\ell(x,y \mid \Lambda) = \gamma^n \lambda_2^{n_2} \exp\{-\gamma v_x - \lambda_2 v_z\} \tag{3.5}$$

where n_2 is the number of observation with $x_i>y_i$, $\gamma = \lambda_1+\lambda_3$, $v_x = \sum_{i=1}^{n} \ln\left(\frac{x_i}{\beta}\right)$, $v_z = \sum_{i=1}^{n} \ln\left(\frac{z_i}{\beta}\right)$ and $z_i = \min(x_i, y_i)$.

Now with the conjugate prior

$$g(\gamma, \lambda_2) = C_4 \ \gamma^{r-1} \lambda_2^{m-1} \exp\{-\gamma v_x' - \lambda_2 v_z'\} \tag{3.6}$$

for γ and λ_2 the posterior density of (γ, λ_2) obtained as

$$f(\gamma, \lambda_2 \mid \underline{x}, \underline{y}) = C_5 \gamma^{N-1} \lambda_2^{M-1} \exp\{-\gamma V_x - \lambda_2 V_z\} \tag{3.7}$$

where $N = n+r$, $N = n_2+m$, $V_x = v_x + v_x'$ and $V_z = v_z + v_z'$.

From (3.7) we obtain the posterior density of R as

$$f(R|\underline{x}, \underline{y}) = [C_6(0)]^{-1} R^{M-1} (1-R)^{N-1} (1-VR)^{-(N+M)}, \quad 0 \le R \le 1, \tag{3.8}$$

where $V = 1 + \frac{V_z}{V_x}$ and

$$C_6(d) = B(N, M+d) \ {}_2F_1(N+M, M+d, M+N+d; V). \tag{3.9}$$

The Bayes estimate R_1 under the asymmetric linear loss function (1.2) is obtained as the solution of the equation

$$[C_6(0)] \int_0^{R_1} R^{M-1}(1-R)^{N-1}(1-VR)^{-(N+M)} dR = \frac{b}{a+b}. \tag{3.10}$$

For the Bayes estimate R_2 under the symmetric loss (1.3) we solve the equation

$$\sum_{j=0}^{2k} \binom{2k}{j} (-1)^j j \delta^{j-1} \frac{C_6(2k-j)}{C_6(0)} = 0. \tag{3.11}$$

When k =1 the Bayes estimate R_2 reduces to

$$R_2 = C_6(1) / C_6(0). \tag{3.12}$$

Finally the Bayes estimate under Linex loss function (1.4) is given by

$$R_3 = \frac{1}{a}\ln\left([C_6(0)]\int_0^1 e^{aR}R^{M-1}(1-R)^{N-1}(1-VR)^{-(N+M)}dR\right)$$

which can be solved numerically for R_3.

4. Monte Carlo Simulation

The estimators obtained in the previous sections are assessed by a numerical study with simulated samples. With different values of the parameters of the model and the hyper parameters of the prior distributions we obtained the estimate and expected loss of the estimators of R based on samples of varying sizes. One such analysis for different value of the hyper parameter is presented in Table I. The values given in the braces are the expected loss corresponding to the estimates given in each cell.

To study the effect of the hyper parameters on the posterior density and its expectations, we adopted the procedure given by Sinha (1980). He suggested that Bayes estimation is robust with respect to its hyper parameters if it leads to a high $\left(\frac{Min}{Max}\right)$ index of the estimate for the varying values of the hyper parameters and is robust with respect to the posterior density if the graphs of the posterior densities for different values of the hyper parameters coincide. For this we calculate the $\left(\frac{Min}{Max}\right)$ index for each of the four hyper parameters by keeping the others fixed for different values of the population parameters for samples of sizes 20, 50, 100 and 200. Also we made several plots of the posterior density. One such plot corresponding to a set of parameter values is presented in figure 1.

In Table II and III, we give the $\left(\frac{Min}{Max}\right)$ index of the estimates for various values of the hyper parameters.

Table 1 Bayes estimate of R

α_3	$\alpha_2\alpha_1$	1	2	3	4	$\frac{Min}{Max}$
1	1	0.512	0.500	0.489	0.478	0.934
	2	0.523	0.511	0.500	0.489	0.934
	3	0.533	0.522	0.511	0.500	0.938
	4	0.543	0.532	0.521	0.510	0.939
	$\frac{Min}{Max}$	0.943	0.940	0.939	0.937	
2	1	0.500	0.489	0.478	0.468	0.936
	2	0.511	0.500	0.489	0.479	0.937
	3	0.522	0.511	0.500	0.490	0.939
	4	0.532	0.521	0.510	0.500	0.940
	$\frac{Min}{Max}$	0.940	0.939	0.937	0.936	
3	1	0.489	0.478	0.468	0.458	0.937
	2	0.500	0.489	0.479	0.469	0.938
	3	0.511	0.500	0.490	0.480	0.939
	4	0.521	0.510	0.500	0.490	0.940
	$\frac{Min}{Max}$	0.939	0.937	0.936	0.935	
4	1	0.478	0.468	0.458	0.449	0.939
	2	0.489	0.479	0.469	0.460	0.941
	3	0.500	0.490	0.480	0.471	0.942
	4	0.510	0.500	0.490	0.481	0.943
	$\frac{Min}{Max}$	0.937	0.936	0.935	0.933	

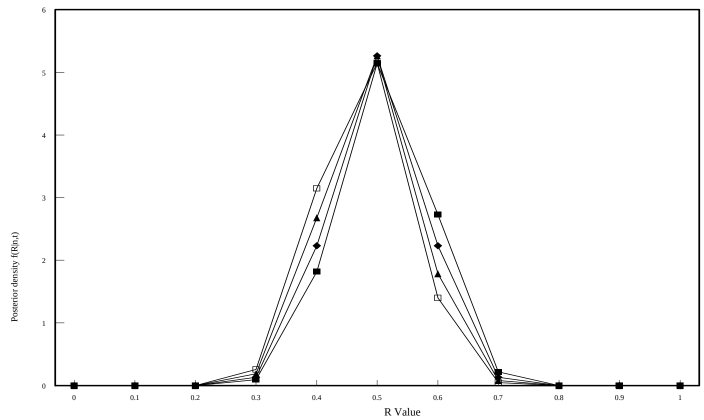

Fig 1. Posterior plot for R, $\alpha_1 = 1(1)4$, $\alpha_2 = 1$, $\alpha_3 = 1$

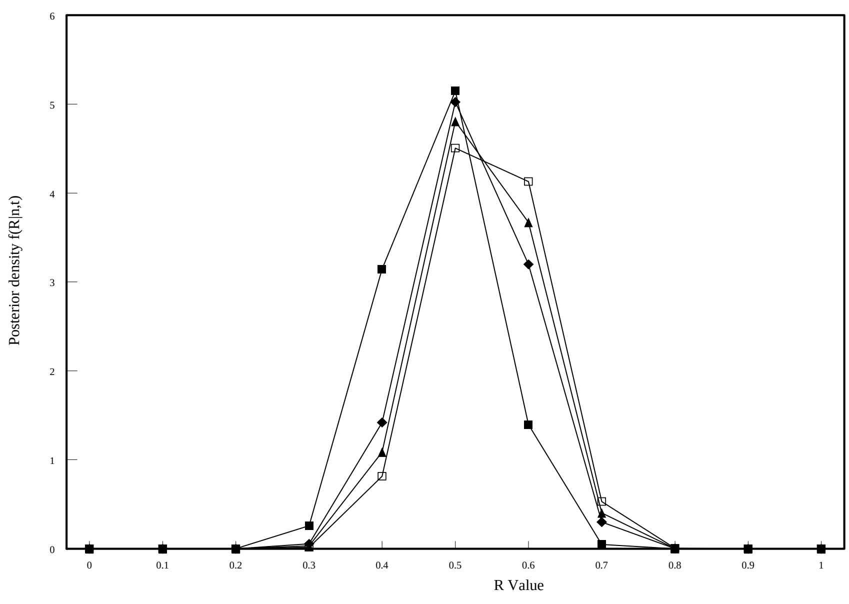

Fig 2. Posterior plot for R, $\alpha_2 = 1(1)4$, $\alpha_1 = 1$, $\alpha_3 = 1$

Finally to assess the performance of the estimates in specific situations, we obtained the bias and mean square error values of the estimators under the Bayesian and the classical frame work using Monte Carlo experiments. For this purpose we generate random samples of sizes 20, 50, 100 and 200 and obtain the above measures empirically using 1000 Monte Carlo runs for different choices of the parameters.

Table 2 Absolute bias and expected loss of $\hat{R}$

α_3	α_1 α_2	1	2	3	4
	1	0.012*	0.000*	0.011	0.022
1	2	0.023*	0.011	0.000	0.011
	3	0.033	0.022	0.011	0.000
	4	0.043	0.032	0.021	0.010
	1	0.000*	0.011	0.022	0.032
2	2	0.011	0.000	0.011	0.022
	3	0.022	0.011	0.000	0.010
	4	0.032	0.021	0.010	0.000
	1	0.011	0.022	0.032	0.042
3	2	0.000	0.011	0.021	0.031
	3	0.011	0.000	0.010	0.020
	4	0.021	0.010	0.000	0.010
	1	0.022	0.032	0.042	0.051
4	2	0.011	0.021	0.031	0.040
	3	0.000	0.010	0.020	0.029
	4	0.010	0.000	0.010	0.019

For those estimates corresponding to * the expected loss is 0.006 and for all others expected loss is 0.005

5. Conclusion

The present paper proposes Bayesian approaches to estimate stress strength reliability of Marshal-Olkin Bivariate Exponential and the Pareto distribution. The estimators are obtained using both symmetric and asymmetric loss functions. Comparisons are made between the different estimators based on a simulation study. The effect of symmetric and asymmetric loss functions was examined and the following were observed:

1. The bias, expected loss and mean square error become smaller as the sample size increase.
2. The Bayes estimate and the posterior density is robust to almost all values of the hyper parameters.
3. When λ_2 increases the bias and MSE dicresses

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