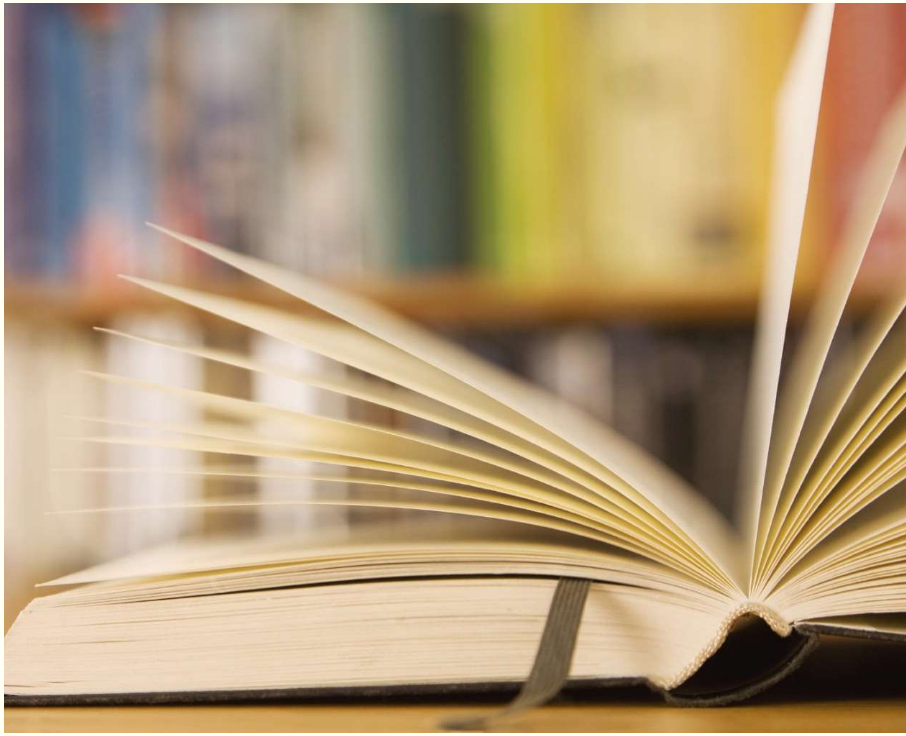
RANDOM EXPERIMENT

Dr.E.S.Jeevanand, Associate Professor
Union Christian College. Aluva



EXPERIMENT

An experiment is an activity which can be repeated in more or less same condition and will have some specific outcome or outcomes. By combining hydrogen with oxygen under certain conditions results in formation of H_2O or tossing a coin and checking whether the face value is Head or Tail are examples of experiments. Generally, experiment are of two types:

1. Deterministic experiment
2. Random Experiment

Deterministic and Random Experiment

Deterministic experiment

An experiment which can be repeated any number of times by anybody and at any place but every time the outcome will be the same provided the initial conditions are kept unchanged. In other words the outcome is determined by the initial conditions. Such experiments are called Deterministic experiment

Random Experiment

An experiment with more than one out comes which can be repeated any number of times under more or less similar condition but the outcomes of which very irregularly from repetition to repetition is called a random experiment. That is for a random experiment there are more than one outcome and the outcome very from repetition to repetition.

Random Experiment

A random experiment is a process characterized by the following properties:

(i) It is performed according to some set of rules,
(ii) It can be repeated arbitrarily often,
(iii) The result of each performance depends on *chance* and cannot be predicted uniquely.

Example: Tossing of a coin

The outcome of a trial can be either head or tail showing up.

Random Experiment

A **random experiment** is an action or process that leads to one of many possible **outcomes**. Examples:

Experiment	Outcomes
Flip a coin	Heads, Tails
Roll a die	Numbers: 1, 2, 3, 4, 5, 6
Exam Marks	Numbers: (0, 100)
Course Grades	F, D, C, B, A
Task completion times	Nonnegative values

Examples

- Selection of a plastic component and verification of its compliance
- Lifetime of a computer
- Number of calls to a communication system during a fixed length interval of time
- The selection of two tools without replacement among a box of three tools.
- The selection of three tools with replacement among a box of two tools.
- Number of accidents in a day
- Number of persons arriving in a queue
- Agriculture production, Industrial production, interest rate, price of the commodities, oil price, gold etc over a period of time

EVENT

Any out come of a random experiment is called an EVENT. An event or event set is a set of possible out come of an experiment.

Consider an experiment of Roll of a Die

Let the experiment be defined as even number turn up. Then the

Event: The outcome is an even number (one of 2, 4, 6), ie E ={2, 4, 6}

Now we can divide this event E into smaller event 2 turn up, 4 turn up and 6 turn up etc. ie E1 = {2}, E2 = {4} and E3={6}.

In this case the Event E is called Compound Event and Event E1, E2 and E3 are called simple events

Sample Space

Simple and Compound Events

An Event which can be divided into smaller events are called compound event and the event which cannot be further divided into smaller events is simple event or sample points.

Sample space

A sample space is the set of all possible simple outcomes (or sample points) of a random experiment.

For the experiment of Roll of a Die, the sample space is

S= {1, 2, 3, 4, 5, 6}.

Note

- Let A be a set. Then the notation $x \in A$ means that x belong to A or the set A contain an element x.
- An event is a non empty subset of the sample space S. We say that the event E has occurred if the observed outcome x is an element of E, that is $x \in E$.
- The Sample space S is called the certain event..
- The null event is an event having no sample point and is denoted by ϕ or $\{\}$. Null event is also called impossible event
- Equally likely events: two or more events are said to be equally likely if any one of them cannot be expected to occur in preference to the other.

Here are some examples

Example 1 *Tossing a coin. The sample space is* $S = \{H, T\}$. $E = \{H\}$ *is an event.*

Example 2 *Tossing a die. The sample space is* $S = \{1, 2, 3, 4, 5, 6\}$. $E = \{2, 4, 6\}$ *is an event, which can be described in words as "the number is even".*

Example 3 *Tossing a coin twice. The sample space is* $S = \{HH, HT, TH, TT\}$. $E = \{HH, HT\}$ *is an event, which can be described in words as "the first toss results in a Heads.*

Example 4 *Tossing a die twice. The sample space is* $S = \{(i, j) : i, j = 1, 2, \ldots, 6\}$, *which contains 36 elements. "The sum of the results of the two toss is equal to 10" is an event.*

Example 5 *Choosing a point from the interval* $(0,1)$. *The sample space is* $S = (0,1)$. $E = (1/3, 1/2)$ *is an event.*

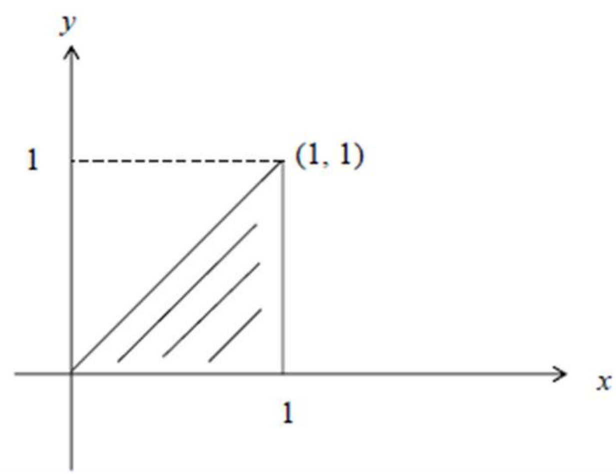
Example 6 *Measuring the lifetime of a lightbulb. The sample space is* $S = [0, \infty)$. $E = [90, \infty)$ *is an event.*

Example 7 *Keeping on tossing a coin until one gets a Heads. The sample space of this experiment is* $S = \{H, TH, TTH, TTTH, \ldots\}$. $E = \{H, TH\}$ *is an event.*

Example 8

Pick a number X at random between zero and one, then pick a number Y at random between zero and X.

$S_3 = \{(x, y): 0 \le y \le x \le 1\}$;
S_3 is a continuous sample space.



Discrete and Continuous Sample Space

- *The sample space of a random experiment is said to be discrete sample space if it contain a finite or countably infinite number of elements*
- That is a sample space is discrete if it contains either

1. A finite number of points or
2. An infinite number of points which may be arranged in some order and counted.

- *The examples 1 to 7 given above are discrete sample space*
- *The sample space of a random experiment is said to be continuous sample space if it contain a continuum of points*
- That is a sample space is continuous if it contains uncountable number of elements. That is they lies in an interval.

The length of a nail produced by a machine, height of a coconut tree, weight of the students in a college are some examples of continuous sample space

Operations on events

- Equality of Events

  Two events E_1 and E_2 are equal, if every element of E_1 belongs to E_2 and every element of E_1 belongs to E_2 and is denoted by $E_1 = E_2$.

- Example If $E_1 = \{1, 2, 3,4\}$ and $E_2 = \{1, 2, 3,4\}$ then we have $E_1 = E_2$.

- Complement of an Events

  For an events E, complement of E is the event containing the elements of the sample space S, which are not belong to E. Complement of E is usually denoted by E' or E^C or $\bar{E}$.

- Example If S= $\{1, 2, 3,4,5,6\}$ and E = $\{1, 2, 3,4\}$ then $E^C = \{5, 6\}$

- Sub Event and Supper Event

  If every element of an events E_1 is an element of the event E_2 we say E_1 is a sub event of E_2 and E_2 is a super event of E_1. and is denoted by $E_1 \subset E_2$

- Example If $E_1 = \{1, 2, 3,4\}$ and $E_2 = \{1, 2, 3,4,5,6\}$ and then $E_1 \subset E_2$.
- **Note:** The null event ϕ is a sub event of any event E and the sample space S is a supper event of any event E.

Operations on events

- Union of Events

  Union of two or more given events is the event whose elements are the elements of at least one of the given events. For two events E_1 and E_1, their union is denoted by $E_1 \cup E_2$ or $E_1 + E_2$

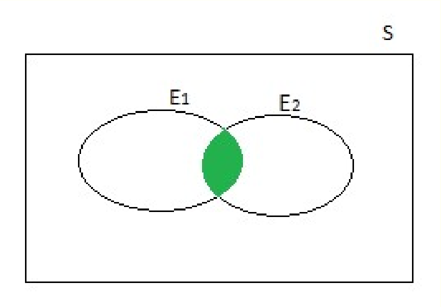
- Example If $E_1 = \{1, 2, 3,4\}$ and $E_2 = \{1, 2, 3,4,5,6\}$ then $E_1 \cup E_2 = \{1, 2, 3,4,5,6\}$

- Intersection of Events

  Intersection of two or more given events is the event containing the elements, which are common to all events under consideration. For two events E_1 and E_1, their intersection is denoted by $E_1 \cap E_2$ or $E_1 E_2$

- Example If $E_1 = \{1, 2, 3,4\}$ and $E_2 = \{2, 3,4,5,6\}$

  then $E_1 \cap E_2 = \{2, 3,4\}$ (In the Diagram it is the green portion.



Operations on events

Union:

a) $E_1 \cup E_2$ occurs means E_1 occurs, or E_2 occurs, or both occur.

b) $E_1 \cup E_2 \cup \cdots \cup E_n$ occurs means that **at least** one of the events $E_1, E_2, \cdots, E_n$ occurs.

Intersection:

a) $E_1 \cap E_2$ occurs means E_1 occurs **and** E_2 occurs.

b) $E_1 \cap E_2 \cap \cdots \cap E_n$ occurs means that **all** of the events $E_1, E_2, \ldots, E_n$ occur.

Complement:

E' occurs means that E **does not occur**.

Some properties of set theoretic operations: on events

Commutativity:

$$E \cup F = F \cup E, \quad E \cap F = F \cap E.$$

Associativity:

$$(E \cup F) \cup G = E \cup (F \cup G), \quad (E \cap F) \cap G = E \cap (F \cap G).$$

Distributivity:

$$(E \cup F) \cap G = (E \cap G) \cup (F \cap G), \quad (E \cap F) \cup G = (E \cup G) \cap (F \cup G).$$

De Morgan's law:

$$(\cup_{i=1}^{n} E_i)^c = \cap_{i=1}^{n} E_i^c, \qquad (\cap_{i=1}^{n} E_i)^c = \cup_{i=1}^{n} E_i^c,$$
$$(\cup_{i=1}^{\infty} E_i)^c = \cap_{i=1}^{\infty} E_i^c, \qquad (\cap_{i=1}^{\infty} E_i)^c = \cup_{i=1}^{\infty} E_i^c.$$

Some properties of set theoretic operations: on events OR Algebraic Laws of Events

DeMorgan Laws:

a)

$$
\begin{aligned}
& (E_1 \cup E_2 \cup \cdots \cup E_n)' \text{ occurs} \\
= \; & \text{none of the events } E_1, E_2, \ldots, E_n \text{ occur} \\
= \; & E_1' \cap E_2' \cap \ldots \cap E_n' \text{ occurs}
\end{aligned}
$$

b)

$$
\begin{aligned}
& (E_1 \cap E_2 \cap \cdots \cap E_n)' \\
= \; & \text{at least one of the events } E_1, E_2, \ldots, E_n \text{ occurs} \\
= \; & E_1' \cup E_2' \cup \ldots \cup E_n' \text{ occurs}
\end{aligned}
$$

Some properties of set theoretic operations: on events

- Identity laws

a. $E \cup \phi = E$ and $E \cap \phi = \phi$

b. $E \cup S = S$ and $E \cap S = E$

- Idempotent laws

$E \cup E = E$ and $E \cap E = E$

- Complement laws

$E \cup E^C = S$

$E \cap E^C = \phi$

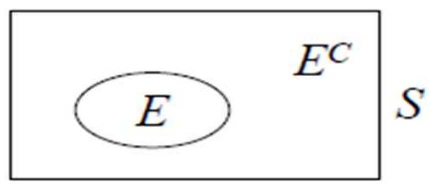
and

$(E^C)^C = E$

Some properties of set theoretic operations: on events

E^C – complement of E in S: defined as the set of elements not in E.

$E^C = \{1, 3, 5\}$, the dice turns up an odd number.



Suppose A and B are events in S, the following events are called derived events

(i)	$A \cup B$	(either A or B or both)
(ii)	$A \cap B$	(both A and B)
(iii)	$A - B$	(A but not B)

Two events A and B are mutually exclusive if both cannot occur simultaneously, that is, $A \cap B = \phi$.

Some properties of set theoretic operations: on events

$\boldsymbol{A \subset B}$

Event A is a subset of event B, then event B will occur whenever event A occurs.

(i) $A \cap B \subset A$ and $A \cap B \subset B$

(ii) $A \subset A \cup B$ and $B \subset A \cup B$

$\boldsymbol{A = B}$ Two events are equal if they contain the same set of outcomes.

Notation

$$\bigcup_{k=1}^{n} A_k = A_1 \cup A_2 \cdots \cup A_n \quad \text{and} \quad \bigcap_{k=1}^{n} A_k = A_1 \cap A_2 \cdots \cap A_n$$

For countably infinite sequence of events, we have

$$\bigcup_{k=1}^{\infty} A_k \qquad \text{and} \qquad \bigcap_{k=1}^{\infty} A_k$$

Mutually Exclusive Events

- Two events which cannot occur together are said to be mutually exclusive or disjoint. If a coin is tossed either head will turn up or tail will turn up. Both head and tail cannot turn up simultaneously. So we say that the events head turn up and tail turn up are mutually exclusive. In other word events of a random experiments are said to be mutually exclusive or disjoint if occurrence of any one event prevents the occurrence of all other events. three mutually exclusive events.

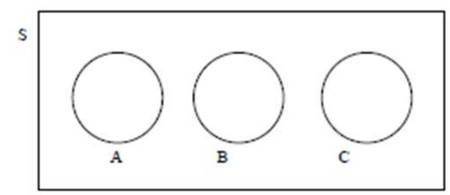
Definition : The events in the sequence $E_1, E_2, \ldots$ are said to be ***mutually exclusive***, if

$$E_i \cap E_j = \emptyset, \text{ for all } i \neq j,$$

where $\emptyset$ represents the empty set.

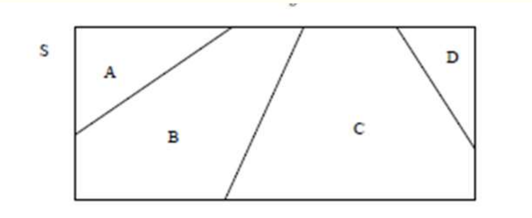
Note: In other words, the events are said to be mutually exclusive if they do not have any outcomes (elements) in common, i.e. they are pairwise disjoint.

three mutually exclusive events.



Exhaustive Events and Partition

- A set of events $E_1, E_2, \ldots, E_k$ are said to be exhaustive events if their union is the whole sample space or in other words every elements of the sample space is an element of at least one of the events $E_1, E_2, \ldots, E_k$. i.e. $E_1 \cup E_2 \cup \cdots \cup E_k = S$.
- Example If S= {1, 2, 3,4,5,6} and E_1 = {1, 2, 3,4} E_2 = {2,4,6} E_3 = {1,3,5} and E_4={5, 6}. Here $E_1 \cup E_2 \cup E_3 \cup E_4 = S$
- A set of events $E_1, E_2, \ldots, E_k$ are mutually exclusive and exhaustive events we say that they form a partition of the sample space
- Example If S= {1, 2, 3,4,5,6} and E_1 = {1, 2,} E_2 = {4} E_3 = {3,5} and E_4={6}. E_1, E_2, E_1, E_2, form a partition of S.



Sigma field of Events

- Any set B of events is called a sigma field of events if it has the follow properties

1. The sample space S and the null event are in B
2. If a finite or countable sequence of events E_1, E_2, ..., are ib B then their union $E_1 \cup E_2 \cup \cdots$ and their intersection then their union $E_1 \cap E_2 \cap \cdots$ are in B.
3. If E is an event in B then its complement E^C is also in B.

- Example If S= {1, 2, 3,4,5,6} and E_1 = {1, 3,5} E_2 = {2,4,6} Then

  B = {S, E_1, E_2, ϕ} = {{1, 2, 3,4,5,6}, {1, 3,5}, {2,4,6}, ϕ} is a sigma field