TESTING EQUALITY OF MEANS OF TWO NORMAL POPULATION

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TESTS FOR THE EQUALITY OF MEANS OF TWO NORMAL POPULATION

Assumption

- There are two independent normal population N(μ_1, σ_1) and N(μ_2, σ_2)
- Two cases when population SD's are known and unknown are considered
- It is also assume that the population standard deviation are equal when they are unknown
- Test is developed for both small and large samples
- Two independent samples of size n_1 and n_2 are taken from each population.
- Let $\bar{x}_1$ and s_1 are the mean and standard deviation of the sample from the first population and $\bar{x}_2$ and s_2 are the mean and standard deviation of the sample from the second population

Summary of Forms for Null and Alternative Hypotheses about equality of Population Means

- The *equality* part of the hypotheses *always* appears in the null hypothesis.
- In general, a hypothesis test about the value of a population mean μ must take one of the following three forms (*where* μ_0 *is the hypothesized value of the population mean*).

H_0: $\mu_1 = \mu_2$ H_a: $\mu_1 < \mu_2$	H_0: $\mu_1 = \mu_2$ H_a: $\mu_1 > \mu_2$	H_0: $\mu_1 = \mu_2$ H_a: $\mu_1 \neq \mu_2$
One-tailed (lower-tail)	One-tailed (upper-tail)	Two-tailed

When population SD's Known

- Use the equation to find the ***actual Z***--Z statistics.
- Use the *Z table (Normal table)* to find the ***critical Z value,*** *and*

The rejection rule is:

- Lower tail: Reject H_0 if Actual $z \leq$ Critical $-z_\alpha$
- Upper tail: Reject H_0 if Actual $z \geq$ Critical z_α
- Two tail: Reject H_0 if Actual $|z| \geq$ Critical $z_{\alpha/2}$

Equation for finding the actual Z value:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

When population SD's Unknown and Sample is large

- Use the equation to find the *actual Z*--Z statistics.
- Use the *Z table (Normal table)* to find the ***critical Z value***, *and*

- ***The rejection rule is:***
  - Lower tail: Reject H_0 if Actual $z \leq$ Critical $-z_\alpha$
  - Upper tail: Reject H_0 if Actual $z \geq$ Critical z_α
  - Two tail: Reject H_0 if Actual $|z| \geq$ Critical $z_{\alpha/2}$

Equation for finding the actual Z value:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

When population SD's Unknown and Sample is small

- Use the equation to find the *actual t* – t statistics.
- Use the *t table with* $n_1 + n_2 - 2$ degrees of freedom to find the ***critical t value.***

The rejection rule is:

- Lower tail: Reject H_0 if Actual $t \leq$ Critical $-t_\alpha$
- Upper tail: Reject H_0 if Actual $t \geq$ Critical t_α
- Two tail: Reject H_0 if Actual $|t| \geq$ Critical $t_{\alpha/2}$

Equation for finding the actual t value:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Steps of Hypothesis Testing

- Step 1. Develop the null and alternative hypotheses
- Step 2. Specify α and n.
- Step 3. Compute *critical Z* and *actual Z values*.

  Step 4. make conclusion:

EXAMPLE - 1

- A sample of 400 men from South India has a mean hight of 65.85 inches and a SD of 2.50 inches which a sample of 100 men from North India has a mean height of 66.20 inches with a standard deviation 2.52 inches. Do the data indicate that North Indian are on the average taller than South Indian?
- Given n_1= 400 $\bar{x}_1$=65.85 s_1= 2.50
- n_2= 100 $\bar{x}_2$=66.20 s_2= 2.52

- Let μ_1 and μ_2 be the mean height of South Indian and North Indian. Then we have to test the hypothesis.
- $H_0{:}\mu_1 = \mu_2$ $H_1{:}\mu_1 < \mu_2$
- Since the sample is large the test statistics is
- $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{65.85 - 6\ \ .20}{\sqrt{\frac{2.5^2}{400} + \frac{2.5^2}{100}}}$ = -1.24 >-1.65
- not significant . So the data does not indicate that North Indian are on the average taller than South Indian

EXAMPLE - 2

- Random samples of sizes 500 and 400 are found to have means 11.5 and 10.9 respectively. Can the samples be regarded as random samples drawn from the same population whose sd is 5?
- Given n_1= 500 $\bar{x}_1$=11.5, n_2= 400 $\bar{x}_2$= 10.9 and $\sigma_1 = \sigma_2 = \sigma$= 5.
- Let μ_1 and μ_2 be the mean of the two population. Then we have to test the hypothesis.
- $H_0{:}\mu_1 = \mu_2$ $H_1{:}\mu_1 \neq \mu_2$

- $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \dfrac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$= \frac{11.5 - 10.9}{5\sqrt{\frac{1}{500} + \frac{1}{400}}} = 1.79 < 1.96.$$

- not significant .The samples may be regarded as random samples drawn from the same population.

EXAMPLE -3

- Samples of size 10 and 14 were taken from two normal populations with standard deviations 3.5 and 5.2 and the sample means were found to be 20.3 and 18.6. Test whether the means of two population are the same at 5% level?
- Given n_1= 10 $\bar{x}_1$=20.3, n_2= 14 $\bar{x}_2$= 18.6

  σ_1 =3.5 and σ_2= 5.2
- Let μ_1 and μ_2 be the mean of the two population. Then we have to test the hypothesis.
- H_0:$\mu_1 = \mu_2$ H_1:$\mu_1 \neq \mu_2$

- $Z = \frac{20.3-18.6}{\sqrt{\frac{3.5^2}{10}+\frac{5.2^2}{14}}} = 0.96 < 1.96.$
- not significant . The samples may be regarded as random samples drawn from the same population.

EXAMPLE -4

- Samples of size 10 men and another sample of size 12 of women have mean IQ's 101 and 98 respectively. Assuming that the IQ's of men and women are independently and normally distributed with mean μ_1 and μ_2 and sd's 4 and 3 examine whether men are on the average more intelligent than women at 5% level.
- Given $n_1 = 10$ $\bar{x}_1 = 101$, $n_2 = 12$ $\bar{x}_2 = 98$

  $\sigma_1 = 4$ and $\sigma_2 = 3$.
- we have to test the hypothesis $H_0: \mu_1 = \mu_2$

  $H_1: \mu_1 > \mu_2$

- $$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

  $$= \frac{101-}{\sqrt{\frac{4^2}{10} + \frac{3^2}{12}}} = 1.89 > 1.65.$$

- Here the calculated value of Z is grater than the tabled value and so the test is significant .We reject the hypothesis that intelligence quotients of men and women are equal and accept the alternative hypothesis that men have higher IQ than women.

EXAMPLE -5

- 12 rats were given a high protein diet and another set of 7 rats were given low protein diet. The gain in weight in gms are given below

  $n_1 = 12 \quad \bar{x}_1 = 11.5 \quad s_1^2 = 4.41$

  $n_2 = 7 \quad \bar{x}_2 = 9.7 \quad s_2^2 = 3.35$

- Examine whether the high protein diet is superior to the law protein diet.
- Let μ_1 and μ_2 be the mean of the two diet Then we have to test the hypothesis.
- $H_0: \mu_1 = \mu_2 \; H_1: \mu_1 > \mu_2$
- Here the population SD's are not known. So we assume that population SD's are equal. The test Statistics is

- $$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{11.5 - 9.7}{\sqrt{\frac{53+2 \quad .42}{12+7-}\left(\frac{1}{12} + \frac{1}{7}\right)}} = 1.78.$$

From the t table with 17 degrees of freedom $t_\alpha = 1.74$. Since the calculated value is greater than the tabulated value the test is significant . So the hypothesis is reject the hypothesis. So the diet is not superior.

The End

For any doubt please contact

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