TESTING THE HYPOTHESIS OF POPULATION PROPORTION

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TESTING THE HYPOTHESIS OF POPULATION PROPORTION

Consider a population, the elements of which may be classified into two categories; those possessing a particular characteristic and those not possessing it. For example human population which may be classified into males and females or those suffering from a particular disease and those not suffering from it or smokers and non smokers or any thing like that. Let p denote the proportion of elements of the population which possess the characteristic and q=1-p denote the proportion of elements of the population which does not possess the characteristic. In this case we consider the problem of testing the hypothesis concerning the value of p. Two problem usually crop up in this connection (1) to test $\boldsymbol{H_0}:\boldsymbol{p} = \boldsymbol{p_0}$ where $\boldsymbol{p_0}$ is a value suggested theoretically or from experience and (2) to test whether the proportion of elements possessing the characteristic is the same in two or population i.e. $\boldsymbol{H_0}:\boldsymbol{p_1} = \boldsymbol{p_2}$. This test is applicable only for large sample i.e. when n>50.

Testing the hypothesis of population proportion has specified value $H_0{:}p = p_0$

- The *equality* part of the hypotheses *always* appears in the null hypothesis.

- The alternative hypothesis are.

$H_0{:}p = p_0$ $H_a{:}p < p_0$	$H_0{:}p = p_0$ $H_a{:}p > p_0$	$H_0{:}p = p_0$ $H_a{:}p \neq p_0$
One-tailed (lower-tail)	One-tailed (upper-tail)	Two-tailed

TESTING THE HYPOTHESIS OF POPULATION PROPORTION HAS SPECIFIED VALUE $\boldsymbol{H_0{:}p = p_0}$

- Let a sample of size n be taken and let x be the number of units possessing the characteristic. To test the hypothesis $\boldsymbol{H_0{:}p = p_0}$ we use the Z statistics
- $z = \dfrac{\frac{x}{n} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$ which has normal distribution.

The rejection rule is:

- Lower tail: Reject H_0 if Actual $z \leq$ Critical $-z_\alpha$
- Upper tail: Reject H_0 if Actual $z \geq$ Critical z_α
- Two tail: Reject H_0 if Actual $|z| \geq$ Critical $z_{\alpha/2}$

EXAMPLE – 1

- In a sample of 100 people the number of those suffering from TB was found to be 5. Does this contradict the assumption that the proportion of TB patients in the whole population is less than 0.04
- we have to test the hypothesis.

  $H_0{:}p = 0.04 \;\; H_1{:}p < 0.04$
- Given n = 100 x = 5 p_0= 0.04 q_0= 0.96
- $z = \frac{\frac{x}{n} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{\frac{5}{100} - 0.04}{\sqrt{\frac{0.04 \times 0.96}{100}}}$ = 0.51> -1.645 not significant . So there is no reason to reject the hypothesis. So we conclude that the proportion of TB patients in the whole population is equal to 0.04

EXAMPLE – 2

- The records of certain hospital showed the birth of 723 males and 617 females in a certain week. Do these conform to the hypothesis that the sexes are born in equal proportions.
- Let p be the proportion of male children. We have to test the hypothesis $H_0{:}p = 1/2 \;\; H_1{:}p \neq 1/2$.
- n = 723+617=1340 x = 723 p_0= 0.5 q_0= 0.5
- $z = \frac{\frac{x}{n} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{\frac{723}{1340} - 0.5}{\sqrt{\frac{0.5 \times 0.5}{1340}}}$ = 2.896> 1.96
- significant and we reject the null hypothesis. So the data does not conform to the hypothesis, the both sexes are born in equal proportion.

Summary of Forms for Null and Alternative Hypotheses about equality of Proportion

- The *equality* part of the hypotheses *always* appears in the null hypothesis.

- The alternative hypothesis are

One-tailed (lower-tail)	One-tailed (upper-tail)	Two-tailed
$H_0: p_1 = p_2$ $H_a: p_1 < p_2$	$H_0: p_1 = p_2$ $H_a: p_1 > p_2$	$H_0: p_1 = p_2$ $H_a: p_1 \neq p_2$

TESTING THE EQUALITY OF PROPORTION IN TWO POPULATION $H_0: p_1 = p_2$.

- Let two sample of size n_1 and n_2 be taken from each population and let x_1 and x_2 be the number of units possessing the characteristics. To test the hypothesis $H_0: p_1 = p_2$ we use the Z statistics

- $z = \dfrac{\frac{x_1}{n_1} - \frac{x_2}{n_2}}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ which has normal distribution where P = $\dfrac{x_1 + x_2}{n_1 + n_2}$. Q=1-P.

The rejection rule is:

- Lower tail: Reject H_0 if Actual $z \leq$ Critical $-z_\alpha$
- Upper tail: Reject H_0 if Actual $z \geq$ Critical z_α
- Two tail: Reject H_0 if Actual $|z| \geq$ Critical $z_{\alpha/2}$

EXAMPLE -3

- In a Samples of 600 men from a certain city 400 are found to be smokers. In 900 from another city 450 are smokers. Do the data indicate that the cities are significantly differs as far as smoking habit of the people are concerned.
- Given n_1= 600 x_1=400, n_2= 900 x_2= 450

  P = $\frac{400+4}{600+900}$ = 0.56 and Q= 1-P = 0.44
- $H_0{:}p_1 = p_2 \; H_1{:}p_1 \neq p_2$
- $z = \frac{\frac{x_1}{n_1} - \frac{x_2}{n_2}}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{\frac{400}{600} - \frac{450}{900}}{\sqrt{0.56 \times 0.44\left(\frac{1}{600}+\frac{1}{900}\right)}}$=6.38

Here |z|> $z_{\alpha/2}$ = 1.96 is significant. So the cities are significantly different as far as smoking habits of people are concerned.

EXAMPLE -4

In two colleges affiliated to a university 46 out of 200 and 48 out of 250 candidates failed in an examination. If the percentage of failure in the university is 18% examine whether the two college differs.

- Given n_1= 200 x_1=46, n_2= 250 x_2= 48. Also given

  P = $\frac{18}{10}$ = 0.18 and Q= 1-P = 0.82
- $H_0{:}p_1 = p_2 \; H_1{:}p_1 \neq p_2$
- $z = \frac{\frac{x_1}{n_1} - \frac{x_2}{n_2}}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{\frac{46}{200} - \frac{48}{25}}{\sqrt{0.18 \times 0.82\left(\frac{1}{200}+\frac{1}{25}\right)}}$=1.043

Here |z|< $z_{\alpha/2}$ = 1.96 is not significant. So the two collges do not differ significantly.

The End

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