TESTING THE HYPOTHESIS FOR STANDARD DEVIATION OF NORMAL POPULATION

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In this case we consider the testing of hypothesis related to the standard deviation of the normal population. Two problem usually crop up in this connection (1) to test $\boldsymbol{H_0{:}\sigma = \sigma_0}$ where $\boldsymbol{\sigma_0}$ is a value suggested theoretically or from experience and (2) to test whether the standard deviation of two normal population are the same or not. i.e. $\boldsymbol{H_0{:}\sigma_1 = \sigma_2}$. This test are developed for both small and large sample.

Testing the hypothesis of Standard deviation of a normal population has specified value $\boldsymbol{H_0 : \sigma = \sigma_0}$

- The *equality* part of the hypotheses *always* appears in the null hypothesis. Note that since $\boldsymbol{\sigma}$ cannot take negative values the alternate hypothesis is always $\boldsymbol{H_1 : \sigma > \sigma_0}$

$$H_0 : \sigma = \sigma_0$$

$$H_a : \sigma > \sigma_0$$

TESTING THE HYPOTHESIS OF STANDARD DEVIATION OF A NORMAL POPULATION HAS SPECIFIED VALUE $\boldsymbol{H_0 : \sigma = \sigma_0}$ *IN THE CASE OF SMALL SAMPLE*

- Let a sample of size n be taken from the population and let s be the sample standard deviation. To test the hypothesis $\boldsymbol{H_0 : \sigma = \sigma_0}$ we use the χ^2 statistics.

- $\chi^2 = \dfrac{ns^2}{\sigma_0^2}$ which has chi-square distribution with n-1 degrees of freedom.

The rejection rule is:

  - Reject H_0 if Actual $\boldsymbol{\chi^2} \geq$ Critical χ^2_α

EXAMPLE – 1

- The standard deviation of sample of size 15 from a normal population was found to be 7. Examine whether the hypothesis that the SD is 7.6 is acceptable.
- we have to test the hypothesis.

  $H_0{:}\sigma = 7.6 \;\; H_1{:}\sigma > 7.6$
- Given n = 15 s = 7 σ_0= 7.6
- $\chi^2 = \frac{ns^2}{\sigma_0^2} = \frac{10\times7^2}{7.6^2}$ = 12.73
- The tabled value χ_α^2 for α=0.05 and 14 degrees of freedom is 23.68. Since the calculated value is less than χ_α^2 it is not significant. So there is no reason to support the hypothesis that $\sigma = 7.6$.

EXAMPLE – 2

- The following figures give the price in rupees of a certain commodity in a sample of selected shops. Assuming the price distribution is normal examine whether the sd of price is 0.30
- 7.41, 7.77, 7.44, 7.40, 7.38, 7.93, 7.58, 8.28, 7.23, 7.52,
- 7.82, 7.71, 7.84, 7.63, 7.68
- n = 15 $\bar{x} = \frac{1}{n}\sum x_i$= 7.64, $s^2 = \frac{1}{n}\sum(x_i - \bar{x})^2 = 0.066$
- σ_0= 0.30 we have to test the hypothesis.

  $H_0{:}\sigma = 0.30 \;\; H_1{:}\sigma > 0.30$
- $\chi^2 = \frac{ns^2}{\sigma_0^2} = \frac{15\times0.066}{0.33^2}$ = 11
- The tabled value χ_α^2 =23.68. Since the calculated value is less than χ_α^2 it is not significant. So there is no reason to support the hypothesis that $\sigma = 0.30$.

TESTING THE HYPOTHESIS OF STANDARD DEVIATION OF A NORMAL POPULATION HAS SPECIFIED VALUE $H_0{:}\sigma = \sigma_0$ *IN THE CASE OF LARGE SAMPLE*

- Let a sample of size n be taken from the population and let s be the sample standard deviation. To test the hypothesis $H_0{:}\sigma = \sigma_0$ we use the z statistics.
- $z = \frac{(s-\sigma_0)\sqrt{2n}}{\sigma_0}$ which has Standard Normal distribution.

The rejection rule is:

- Reject H_0 if Actual $z \geq$ Critical z_α

EXAMPLE -3

- Sd of a sample of size 50 from a normal population is 3.5. Examine whether the sample was taken from population with SD 3.2 at 5% level of significant.
- Given n=50 s= 3.5 and σ_0=3.2,
- $H_0 : \sigma = 3.2 \;\; H_1 : \sigma > 3.2$
- $z = \frac{(s-\sigma_0)\sqrt{2n}}{\sigma_0} = \frac{(3.5-3.2)\sqrt{100}}{3.2} = 0.9$
- Here z< z_α = 1.645, so test is not significant. Hence we accept the null hypothesis

$\boldsymbol{H_0 : \sigma = 3.2}$ **.**

EXAMPLE -4

A sample of 50 has standard deviation 10.5. Does this contradict the hypothesis that the population sd is 12.

- Given n=50 s= 10.5 and σ_0=12,
- $H_0 : \sigma = 12 \;\; H_1 : \sigma > 12$
- $z = \frac{(s-\sigma_0)\sqrt{2n}}{\sigma_0} = \frac{(10.5-12)\sqrt{100}}{12} = -1.25$
- Here z< z_α = 1.645, so test is not significant. Hence we accept the null hypothesis

  $\boldsymbol{H_0 : \sigma = 12.}$

That is the sample information Does not contradict the hypothesis that the population sd is 12.

Testing the Equality of Standard deviation of two normal populationsi.e. $\boldsymbol{H_0{:}\sigma_1 = \sigma_2}$.

- The *equality* part of the hypotheses *always* appears in the null hypothesis. In this case the alternate hypothesis is always $\boldsymbol{H_1{:}\sigma_1 \neq \sigma_2}$

$$H_0{:}\sigma_1 = \sigma_2$$

$$H_a{:}\sigma_1 \neq \sigma_2$$

TESTING THE HYPOTHESIS OF STANDARD DEVIATION OF A NORMAL POPULATION HAS SPECIFIED VALUE $H_0 : \sigma_1 = \sigma_2$ *IN THE CASE OF SMALL SAMPLE*

Let s_1 and s_2 be the standard deviation of sample of sizes n_1 and n_2 taken from two normal population. The test statistics suggested is the ration of $\frac{n_1 s_1^2}{n_1 - 1}$ and $\frac{n_2 s_2^2}{n_2 - 1}$, the bigger of the two being taken as the numerator. The ratio follows F distribution with (n_1-1, n_2-1) or (n_2-1, n_1-1) degrees of freedom

- *The rejection rule is:*
- Reject H_0 if Actual if F ≥ Critical F_α

- If the first one is bigger then the test statistics is
- $$F_{n_1 - 1, n_2 - 1} = \frac{n_1 (n_2 - 1) s_1^2}{n_2 (n_1 - 1) s_2^2}$$
- If the second one is bigger then the test statistics is
- $$F_{n_2 - 1, n_1 - 1} = \frac{n_2 (n_1 - 1) s_2^2}{n_1 (n_2 - 1) s_1^2}$$

EXAMPLE – 5

- The standard deviation of two sample of sizes 10 and 14 from two normal population was found to be 3.5 and 3.0 respectively. Examine whether the SD's of the population are likely to be equal.
- we have to test the hypothesis.

$$H_0{:}\sigma_1 = \sigma_2 \text{ against } H_1{:}\sigma_1 \neq \sigma_2$$

- Given n_1= 10 s_1= 3.5 n_2= 14 s_2= 3
- $\frac{n_1 s_1^2}{n_1-1}$ =13.7 and $\frac{n_2 s_2^2}{n_2-1}$ = 9.69 Since first one is larger
- $F_{n_1-1,n_2-1} = \frac{n_1(n_2-1)s_1^2}{n_2(n_1-1)s_2^2} = \frac{13.7}{9.69}$ =1.40
- The tabled value F_α for α=0.05 and (9,13) degrees of freedom is 2.71. Since the calculated value is less than tabled value test is not significant. So the population SD's are equal.

EXAMPLE – 6

- The variance of two sample of sizes 5 and 6 from two normal population was found to be 13.20 and 17.67 respectively. Examine whether the SD's of the population are likely to be equal.

$$H_0{:}\sigma_1 = \sigma_2 \text{ against } H_1{:}\sigma_1 \neq \sigma_2$$

- Given n_1= 5 s_1^2 = 13.20 n_2= 6 s_1^2 = 17.67
- $\frac{n_1 s_1^2}{n_1-1}$ =16.5 and $\frac{n_2 s_2^2}{n_2-1}$ = 21.2 Since second one is larger
- $F_{n_2-1,n_1-1} = \frac{n_2(n_1-1)s_2^2}{n_1(n_2-1)s_1^2} = \frac{21.2}{16.5}$ =1.29
- The tabled value F_α for α=0.05 and (5,4) degrees of freedom is 6.26. Since the calculated value is less than tabled value test is not significant. So the population SD's are equal.

TESTING THE HYPOTHESIS OF STANDARD DEVIATION OF A NORMAL POPULATION HAS SPECIFIED VALUE $\boldsymbol{H_0}$:$\boldsymbol{\sigma_1} = \boldsymbol{\sigma_2}$ *IN THE CASE OF LARGE SAMPLE*

- Let s_1 and s_2 be the standard deviation of sample of sizes n_1 and n_2 taken from two normal population. To test the hypothesis $\boldsymbol{H_0}$:$\boldsymbol{\sigma_1} = \boldsymbol{\sigma_2}$ we use the z statistics.
- $z = \dfrac{s_1 - s_2}{\sqrt{\dfrac{s_1^2}{2n_1} + \dfrac{s_2^2}{2n_2}}}$ which has Standard Normal distribution.

The rejection rule is:

- Reject H_0 if Actual $|z| \geq$ Critical $z_{\alpha/2}$

EXAMPLE – 7

- A sample of 60 items has SD 5 and another sample of 80 items has SD 4.5 . Can you assert that the two samples belong to the same population
- we have to test the hypothesis. $H_0: \sigma_1 = \sigma_2$ against $H_1: \sigma_1 \neq \sigma_2$
- Given $n_1 = 60$ $s_1 = 5$ $n_2 = 80$ $s_2 = 4.5$
- $z = \frac{s_1 - s_2}{\sqrt{\frac{s_1^2}{2n_1} + \frac{s_2^2}{2n_2}}} = \frac{5 - 4.5}{\sqrt{\frac{25}{120} + \frac{20.25}{160}}} = 0.86 < 1.96$
- Since the calculated value is less than tabled value test is not significant. So the population SD's are equal.

The End

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