PAIRED T TEST

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Paired Test

Paired t test is used to find out the pre and post effect on the same experimental units. For example we wanted to study whether a baby food increases the weight of a children. For this we first choose a set of babies and take the weight of these babies (pre test values) and we will administrate the food to the child for a specified period and take the weight again (post test values). Similarly we can consider the effect of drugs in medical science, effect of counselling in psychology, effect of a method of teaching in education, effect of market intervention in economics etc. Kindly note that this is a small sample test and the observations are of equal numbers. There are two method to evaluate the effects of intervention (1) first method is to use independent sample t test to test the hypothesis. (2) the second one is to use paired t test. In this case the null hypothesis is $\boldsymbol{H_0 : \mu_1 = \mu_2}$ and the alternative hypothesis are either $\boldsymbol{H_1 : \mu_1 > \mu_2}$ if the intervention has negative or reducing effect or $\boldsymbol{H_1 : \mu_1 < \mu_2}$ if the intervention has positive or increasing effect.

Paired t test concept

Let (x_i, y_i) i=1,2,…,n be the paired observations (i.e, the pre test value and post test value of the i^{th} unit).

Now define $u_i = x_i - y_i$, i=1,2,…, n and let $\bar{u}$ be the mean and s_u standard deviation of the differences. Now the hypothesis we wanted to test is $\boldsymbol{H_0{:}\mu_1 = \mu_2}$ which is a two sample hypothesis, here through differencing we convert it into a one sample hypothesis as

$\boldsymbol{H_0{:}\mu_1 - \mu_2}$=0 which can be tested using the one sample t statistics with $\boldsymbol{\mu_0}$=0

$$\boldsymbol{t = \frac{(\bar{u}-\mu_0)\sqrt{n-1}}{s_u} = \frac{(\bar{u}-0)\sqrt{n-1}}{s_u} = \frac{\bar{u}\sqrt{n-1}}{s_u}}$$

which has t distribution with n-1 degrees of freedom.

Note: Some times this is used to test the hypothesis of two sample of equal number of observation.(eg: to compare the effect of two drugs or two type of baby foods etc.)

Steps in paired t test

Let (x_i, y_i) i=1,2,…,n be the paired observations

1. Calculate $u_i = x_i - y_i$, i=1,2,…, n
2. Calculate $\bar{u}$ the mean and s_u standard deviation of u_i
3. Calculate the test statistics value t using $\boldsymbol{t = \frac{\bar{u}\sqrt{n-1}}{s_u}}$
4. For a given α obtain the critical value from the t table corresponding to n-1 degrees of freedom.
5. Make the decision to Accept or reject H_0

The rejection region are

For the hypothesis $\boldsymbol{H_1 : \mu_1 > \mu_2}$ (if the intervention has negative or decreasing effect)

Reject H_0 if Actual $t \geq$ Critical t_α

For the hypothesis $\boldsymbol{H_1 : \mu_1 < \mu_2}$ (if the intervention has positive or increasing effect)

Reject H_0 if Actual $t \leq$ Critical $-t_{\alpha.}$

Example-1

A framer grows crops on two fields A and B. On A he puts Rs.100/- worth of manure per acre and on B Rs.200/- worth manure. The yields per acre for 5 years is given below. Examine we whether costly dressing has resulted in increased yields.

Year	1	2	3	4	5
Yield from A(Rs)	34	28	42	37	44
Yield from B(Rs)	36	33	48	38	50

Let x_i denote the yield from A in the i[th] Year and y_i denote the yield from B in the i[th] Year. Here we have to test the hypothesis $\boldsymbol{H_0{:}\mu_1 = \mu_2}$ against $\boldsymbol{H_1{:}\mu_1 < \mu_2}$. Now

$u_i = x_i - y_i$:	-2	-5	-6	-1	-6

$\bar{u} = \frac{1}{n}\sum x_i = \frac{-20}{5}$ = -4. $s_u^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$ = 4.4 so $s_u = 2.098$.

$\boldsymbol{t = \frac{\bar{u}\sqrt{n-1}}{s_u}} = \frac{-4\sqrt{4}}{2.098}$ = -3.87

The tabled value t_α for α=0.05 and 4 degrees of freedom is 2.26. Since the calculated value of -3.87 is less than -t_α =-2.26 test is significant. So the costly dressing has resulted in increase of yield.

Example 2

A group of 10 children were tested to find out how many digits they could repeat from memory after hearing them once. They were given practice at this test during the next week and were then tested. Is the training effective.

Child	A	B	C	D	E	F	G	H	I	J
Test 1 (x)	6	5	4	7	8	6	7	5	6	8
Test 2 (y)	7	7	6	7	9	6	8	6	6	10

Let x_i denote the test 1 score and y_i denote the test 2 score of i^{th} Student. Here we have to test the hypothesis $\boldsymbol{H_0{:}\mu_1 = \mu_2}$ against $\boldsymbol{H_1{:}\mu_1 < \mu_2}$. Now

$u_i = x_i - y_i$: -1 -2 -2 0 -1 0 -1 -1 0 -2

$\bar{u} = \frac{1}{n}\sum x_i = \frac{-10}{10} = -1$. $s_u = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2} = 0.755$

$\boldsymbol{t} = \frac{\boldsymbol{\bar{u}\sqrt{n-1}}}{\boldsymbol{s_u}} = \frac{-1\sqrt{9}}{0.755} = -3.87$

The tabled value t_α for α=0.05 and 9 degrees of freedom is 1.83. Since the calculated t_α is less than $-t_\alpha$ =-1.83 test is significant. So the training effective.

Example 3

Certain diet was fed to 15 mice and the following increase in weights have been noted. 2.1, 3.2, -1.0, 1.2, -1.3, 1.5, 1.7, 2.5, -3.0, 4.0, 4.2, 1.7, -1.6, 1.2, 1.0. examine whether the diet has any effect on the increase in weight of mice.

Let x_i denote the pre test weight y_i denote post weight of the i[th] mice. Here we have to test the hypothesis $\boldsymbol{H_0}$:$\boldsymbol{\mu_1 = \mu_2}$ against $\boldsymbol{H_1}$:$\boldsymbol{\mu_1 < \mu_2}$. Now we are given the increase in weight i.e $y_i - x_i$. But we define $u_i = x_i - y_i$. So get u_i we have to multiply each of the given observation by -1. That is

u_i -2.1, -3.2, 1.00, -1.2, 1.3, -1.5, -1.7, -2.5, 3.0, -4.0, -4.2, -1.7, 1.6, -1.2, -1.0.

$\bar{u} = \frac{1}{n}\sum x_i = \frac{-17.4}{15}$ = -1.16. $s_u = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2}$ = 0.663

$\boldsymbol{t} = \frac{\boldsymbol{\bar{u}\sqrt{n-1}}}{\boldsymbol{s_u}} = \frac{-1.16\sqrt{14}}{0.663}$ = -6.54

The tabled value t_α for α=0.05 and 9 degrees of freedom is 1.761. Since the calculated t_α is less than -t_α =-1.83 test is significant. So the is effective in the increase in weight of mice.

The End

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