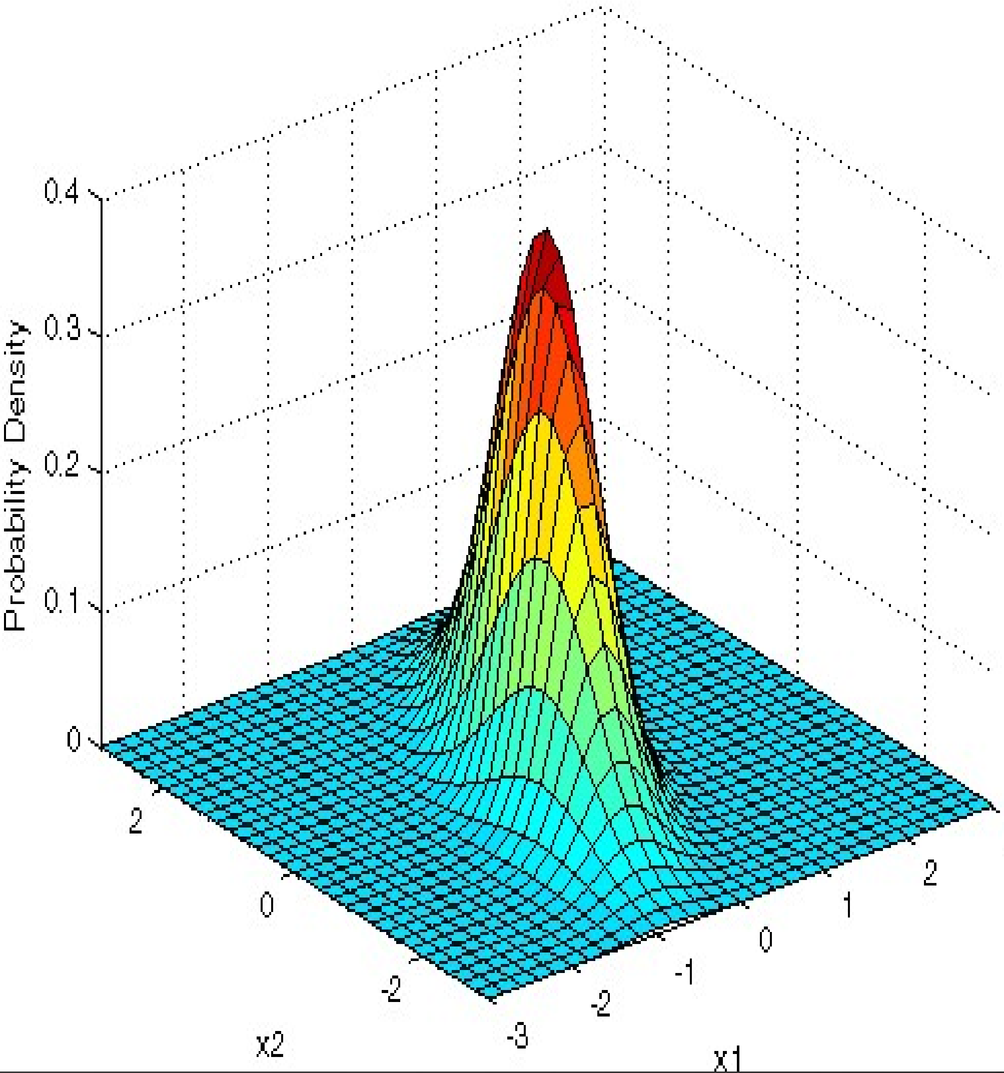


Goal Programming Problem

Example:

A factory can manufacture two products *A* and *B*. The profit on one unit of *A* is Rs 80 and one unit of *B* Rs 40. The maximum demand of *A* is 6 units per week, and of *B* it is 8 units per week. The manufacturer has set up a goal of achieving a profit of Rs 640 per week. Formulate the problem as goal programming and solve it.

Let x_1 = number of units of A to be produced per week

And x_2 = number of units of B to be produced per week

Also let f be the profit

Then $f = 80x_1 + 40x_2$

Goal is 640

f may exceed the goal of 640 or fall short of it

Let $u \geq 0$ denote the shortfall (under achieved)

$v \geq 0$ the excess (over achieved) in profit from the goal.

Three possibilities exist. Goal will be:

1. Exactly achieved ($u = v = 0$)
2. Over-achieved ($u = 0, v > 0$)
3. Under-achieved ($u > 0, v = 0$)

So, only those solutions are acceptable in which at least one of the variables, u or v, is zero or both are zero.

So,

either $f = 640 - u$

or $f = 640 + v$

Combining these two conditions, we get:

$$f + u - v = 640$$

where at least one of the variable, u or v, is zero

To achieve the goal as closely as possible, the objective should be to minimize the deviation from the goal.

Problem formulation

Minimize $F = u + v$

subject to $80x_1 + 40x_2 + u - v = 640$

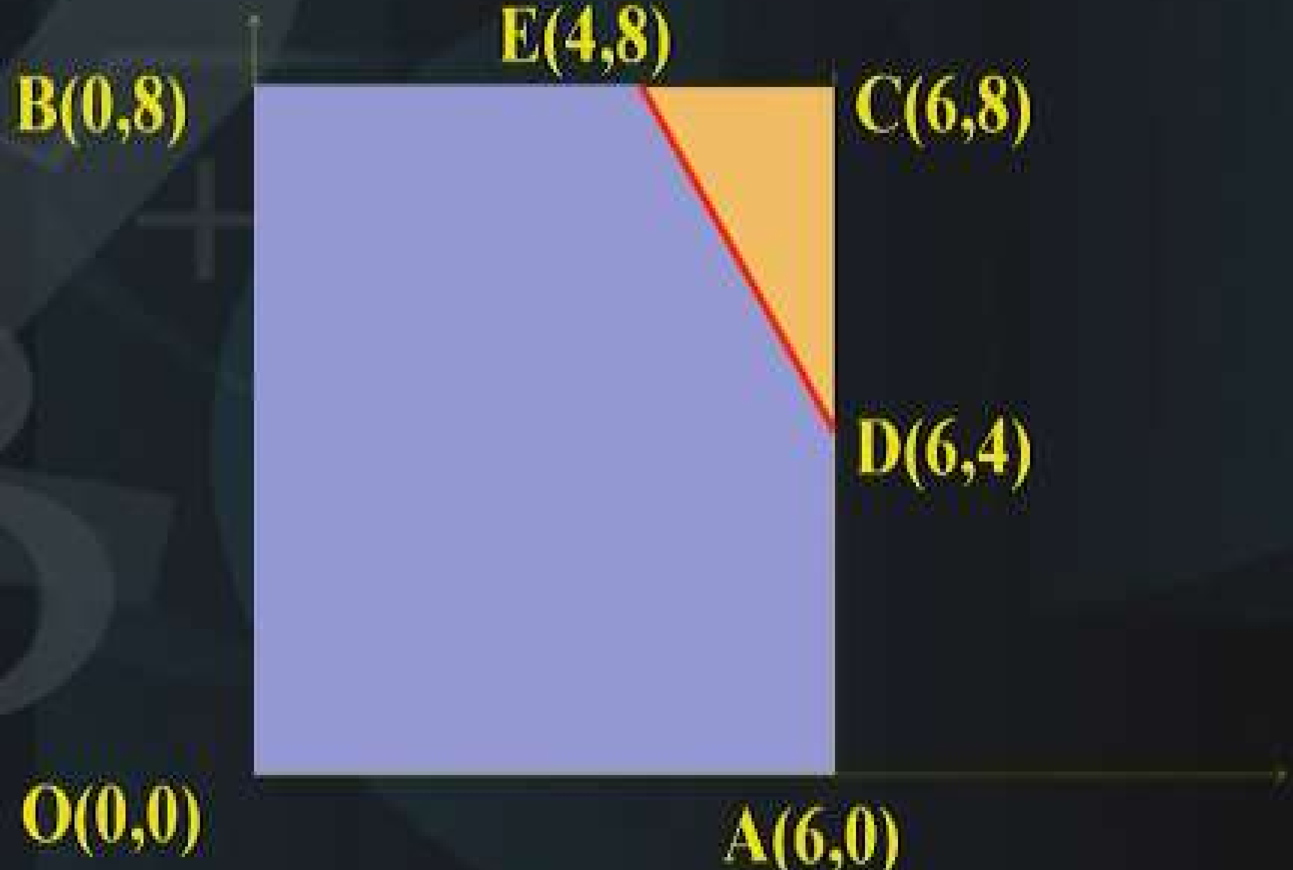
$x_1 \leq 6$

$x_2 \leq 8$

$x_1, x_2, u, v \geq 0$

Such that either u or v or both equal to zero

Graphical solution



Considering the $f = 80x_1 + 40x_2$

On the line DE (shown in red colour), $f = 640$.

Thus, goal can be exactly met by choosing x_1, x_2 such that (x_1, x_2) lies on DE.

For points on DE, $u = v = 0$, giving minimum value of F.

If (x_1,x_2) falls in DEC (shown in peach colour) then profit is over and above 640 (u=0, v > 0).

If (x_1,x_2) falls in OADEB (shown in purple colour) then profit is less than 640 (u > 0, v = 0).

Solution Using Simplex method

Minimize $F = u + v$

s. t. $80x_1 + 40x_2 + u - v = 640$

$x_1 \leq 6$

$x_2 \leq 8$

$x_1, x_2, u, v \geq 0$

Such that either u or v or both equal to zero

LP in standard form

Minimize $F = u + v$

s. t. $80x_1 + 40x_2 + u - v = 640$

$x_1 + x_3 = 6$

$x_2 + x_4 = 8$

$x_1, x_2, u, v, x_3, x_4 \geq 0$

Such that either u or v or both equal to zero

Caution

Perform the simplex calculations making sure that at any iteration, u and v do not enter the basis together. Either u is in the basis or v is in the basis or none of them is in the basis.

		0	0	-1	-1	0	0	
C_B	Basis	x_1	x_2	u	v	x_3	x_4	RHS
-1	u	80	40	1	-1	0	0	640
0	x_3	1	0	0	0	1	0	6
0	x_4	0	1	0	0	0	1	8
		80	40	0	-2	0	0	-640
-1	u	0	40	1	-1	-80	0	160
0	x_1	1	0	0	0	1	0	6
0	x_4	0	1	0	0	0	1	8
		0	40	0	-2	-80	0	=-160
0	x_2	0	1	1/40	-1/40	-2	0	4
0	x_1	1	0	0	0	1	0	6
0	x_4	0	0	-1/40	1/40	2	1	4
		0	0	-1	-1	0	0	z=0

Solution is $x_1 = 6$ and $x_2 = 4$, with $z = 0$.

This corresponds to the point D(6, 4) in the graphical solution

But the optimal table indicates that this is the case of multiple solutions.

So perform another iteration.

		0	0	-1	-1	0	0	
C_B	Basis	x_1	x_2	u	v	x_3	x_4	RHS
0	x_2	0	1	1/40	-1/40	-2	0	4
0	x_1	1	0	0	0	1	0	6
0	x_4	0	0	-1/40	1/40	2	1	4
		0	0	-1	-1	0	0	z=0
0	x_2	0	1	0	0	0	1	8
0	x_1	1	0	1/80	-1/80	0	-1/2	4
0	x_4	0	0	-1/80	1/80	1	1/2	2
		0	0	-1	-1	0	0	z=0

Alternate solution is:

$x_1 = 6$ and $x_2 = 4$, with $z = 0$.

This corresponds to the point E(4, 8) in the graphical solution.

So, the problem has multiple solutions, name ly all points on the line segment joining D and E.

On the line segment DE, $u = 0$ and $v = 0$.

So the goal is exactly achieved on all points of DE.

Def: linear goal programming

Minimize $F = u + v$

subject to $f(X) + u - v = g$

$$\sum_{j=1}^{n} a_{ij} x_j = b_i \quad i = 1, 2, \ldots, m,$$

$$u, v, x_j \geq 0, \quad j = 1, 2, \ldots, n,$$

with either u or v or both equal to zero.

Where u is the underachievement

v is the overachievement and

g is the goal.

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