

Operations Research

During World War II, the military management in England called on a team of scientist to study the strategic and technical problem of air and land defence. This team of scientists included physicists, mathematicians, statisticians, engineers, psychologists and many others. The objective was to determine the most effective utilization of limited military resources. This group of scientists forms the first OR team. The name operations research (or operational research) was apparently coined because the team was carrying out research on military operations. Immediately after the war, the success of military team attracted the attention of industrial managers who were seeking solution to their problem. As a result OR soon spread from military to Government, industrial, social and economic planning.

Definition of Operations Research

Operations research is a scientific methods, techniques and tools of providing executive department with a quantitative basis for decision regarding the operations under their control.

Characteristic of Operations Research

The essential characteristic of operations research are

1. OR study the system as a whole. i.e. OR is system (or Executive) Orientation.
2. OR uses interdisciplinary teams
3. The application of Scientific Methods
4. OR cannot cover new problems arises in later stage

Phases of Operations Research

The different phases of the Operations Research approach are

1. Formulation the Problem
2. Construction of a model to represent the system under study
3. Deriving a solution from the model
4. Testing the model and the solution derived from it.
5. Establishing the control over solution.
6. Establishing the solution to work, i.e. implementation.

Models of Operations Research

The main models of Operations Research are

- | | |
|--|-------------------------------|
| 1. Mathematical Programming techniques | 8. Queuing models |
| 2. Inventory Models | 9. Dynamic Programming models |
| 3. Allocation Model | 10. Simulation techniques |
| 4. Transportation Problems | 11. Decision theory |
| 5. Sequencing models | 12. Replacement models |
| 6. Routing models | 13. Heuristic models |
| 7. Competitive models | 14. Combined methods |

Linear Programming Problem

The mathematical programming problems in general deals with determining optimal allocation of limited resources to meet given objectives. The resources may be materials, men, machines etc. The main branches of mathematical programming are Linear Programming (LPP), Non-Linear Programming (NLPP), Integer Programming (IPP) and Goal Programming. A Linear Programming Problem deals with the optimization (maximization or minimization) of a function of variables known as *Objective function*, subject to a set of linear equations and/or in equations known as restrictions or constraints. The objective function may be profit, cost, production capacity or any other measure of effectiveness, which is to be obtained in the best possible or optimal manner. The restrictions may be imposed by different sources such as market demand, production processes and equipment, storage capacity, raw material availability etc. By linearity is meant a mathematical expression (equations) in which the variables do not have powers.

Thus Linear Programming may be defined as a method to obtain an optimum solution to a linear objective function subject to a set of linear constraints as defined above.

The requirement of a Linear Programming Problem

Linear programming problem can be used for optimization problems if the following conditions are satisfied

1. There must be a well defined objective function (in terms of profit, cost or quantity produced) which is to be maximized or minimized and which can be expressed as a linear function of decision variables.
2. There must be restrictions on the amount or extent of attainment of the objective and these restrictions must be capable of being expressed as linear equalities or inequalities in terms of variables.
3. There must be alternative course of action. For example, given a product may be processed by two different machines and problem may be as to how much of the product to allocate to which machine.
4. Another necessary requirement is that the decision variables should be interrelated and non-negative. The non-negativity condition shows that linear programming deals with real life situations for which negative quantities are illogical.
5. Finally the resources must be in limited supply.

General form of a Linear Programming Problem

$$\text{Optimize } Z = c_1x_1 + c_2x_2 + \dots + c_nx_n.$$

Subject to

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = (\text{or } \geq \text{or } \leq) b_1.$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = (\text{or } \geq \text{or } \leq) b_2$$

...

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = (\text{or } \geq \text{or } \leq) b_m.$$

$$x_1, x_2, \dots, x_n \geq 0.$$

Advantages of a Linear Programming Problem

1. It provides the insight and perspectives into problem environment related with a multi dimensional phenomenon. This generally results in clear picture of the true problem.
2. It makes a scientific and mathematical analysis of the problem situations. It also considers all possible aspects and remedies associates with the problem.
3. It gives an opportunity to the decision maker to formulate his strategies consistent with the constraints and the objectives.
4. It deals with changing situations. Once a plan is arrived through a Linear Programming, it can be revaluated for changing conditions.

Formulation of Linear Programming Problem.

The main steps in formulation of a Linear Programming Problem are

Steps

1. Study the situation and find the key-decision to be made. (i.e. to identify whether the problem is minimization or maximization)
2. Assume symbols for variable quantities or identify the decision variables noticed in step 1.
3. Express the feasible alternative mathematically in terms of variables. Feasible alternative are those which are physically, economically and financially possible.
4. Mention the objective quantity and expressed it as a linear function of the variables.
5. Express the influence factors or restriction (constrains) into mathematical linear functions.

Example –1

A firm produces three products. Three products are processed on three different machines. The time required to manufacture one unit of each of three products and the daily capacity of the tree machine are given in the table below

Machine	Time per unit (minutes)			Machine Capacity
	Product 1	Product 2	Product 3	
M ₁	2	3	2	440
M ₂	4	-	3	470
M ₃	2	5	-	430

It is required to determine the daily number of units to be manufactures for each product. The profit per unit for product 1, 2 and 3 are Rs.4, Rs.3 and Rs.6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate the LPP

Step 1: The Key –decision to be made is to determine the number of each product 1, 2 and 3 to be produced so has to maximize profit.

Step 2: The decision variables are x_1 , x_2 and x_3 , which represent the number of units of product 1, 2 and 3 produced daily.

Step 3: The feasible alternatives are the set of variables x_1, x_2 and x_3 , where $x_1, x_2, x_3, \geq 0$.

Step 4: The objective function is Maximize $Z = 4x_1 + 3x_2 + 6x_3$

Step 5: The constraints are

$$\begin{aligned} 2x_1 + 3x_2 + 2x_3 &\leq 440 \\ 4x_1 + 0x_2 + 3x_3 &\leq 470 \\ 2x_1 + 5x_2 + 0x_3 &\leq 430. \end{aligned}$$

Thus the LPP is

$$\text{Maximize } Z = 4x_1 + 3x_2 + 6x_3$$

Subject to

$$\begin{aligned} 2x_1 + 3x_2 + 2x_3 &\leq 440 \\ 4x_1 + 3x_3 &\leq 470 \\ 2x_1 + 5x_2 &\leq 430. \\ x_1, x_2, x_3, &\geq 0. \end{aligned}$$

Example –2

An advertising company wishes to plan its advertising strategy in three different media-television, radio and magazines. The purpose of advertising is to reach as large a number of potential customers as possible. Following data has obtained from market survey

	Television	Radio	Magazine I	Magazine II
Cost of advertising units	Rs.30,000	Rs.20,000	Rs.15,000	Rs.10,000
No of potential customers reached per unit	2,00,000	6,00,000	1,50,000	1,00,000
No of female customers reached per unit	1,50,000	4,00,000	70,000	50,000

The company wants to spend not more than Rs. 450,000 on advertising. Following are the further requirement that must be met:

1. at least 1 million exposures take place among female customers
2. advertising on magazine be limited to Rs.1,50,000
3. at least 3 advertising units be brought on magazine I and 1 units in magazine II and
4. the number of advertising units on television and radio should each be between 5 and 10.

Formulate the LPP

Step 1: The Key –decision to be made is to determine the number of advertising units to be brought in television, radio and magazines so has to reach as large a number of potential customers

Step 2: The decision variables are x_1, x_2, x_3 and x_4 , which represent the number of advertising units in television, radio, magazine I and magazine II.

Step 3: The feasible alternatives are the set of variables x_1, x_2, x_3 and x_4 , where $x_1, x_2, x_3, x_4 \geq 0$.

Step 4: The objective function is Maximize $Z = 10^5 (2x_1 + 6x_2 + 1.5x_3 + 1x_4)$

Step 5: The constraints are

$$\begin{aligned} 30,000x_1 + 20,000x_2 + 15,000x_3 + 10,000x_4 &\leq 4,50,000 \\ 150,000x_1 + 400,000x_2 + 70,000x_3 + 50,000x_4 &\geq 1,00,000 \\ 15,000x_3 + 10,000x_4 &\leq 1,50,000 \\ x_3 &\geq 3 \\ x_4 &\geq 2 \\ 5 &\leq x_1 \leq 10 \\ 5 &\leq x_2 \leq 10 \end{aligned}$$

Thus the LPP is

$$\text{Maximize } Z = 10^5 (2x_1 + 6x_2 + 1.5x_3 + 1x_4)$$

Subject to

$$\begin{aligned} 30x_1 + 20x_2 + 15x_3 + 10x_4 &\leq 450 \\ 15x_1 + 40x_2 + 7x_3 + x_4 &\geq 100 \\ 15x_3 + 10x_4 &\leq 150 \\ x_3 &\geq 3 \\ x_4 &\geq 2 \\ x_1 &\geq 5 \\ x_2 &\geq 5 \\ x_1 &\leq 10 \\ x_2 &\leq 10 \\ x_1, x_2, x_3, x_4 &\geq 0. \end{aligned}$$

Solution of Linear Programming Problem

In general there are two methods to solve the Linear Programming Problem. They are graphical methods if there are only two decision variables and the simplex method. Now we consider some definition related to this

Feasible Solution: A feasible solution to a Linear Programming Problem is the set of the values of the variables which satisfy all the.

Basic feasible solutions: A feasible solution is called a basic feasible solution if it has no more than m positive (non-zero) x_j . In other words, it is a basic solution which also satisfies the non-negativity condition.

Optimal Solution: A basic feasible solution to a Linear Programming Problem is said to be optimum solution if it optimizes the objective function of the problem.

Slack Variables: If the constraints has a \leq then in order to make it an equality we have to add some variable to the left hand side of the constraints. This variables are called slack variables.

Surplus Variables: If the constraints has a sign \geq then in order to make it an equality we have to subtract some variable to the left hand side of the constraints. This variables are called surplus variables.

Graphical Method of Solution to Linear Programming Problem

So far we have learnt how to construct a mathematical model for a linear programming problem. If we can find the values of the decision variables $x_1, x_2, x_3, \dots, x_n$, which can optimize (maximize or minimize) the objective function Z , then we say that these values of x_i are the optimal solution of the Linear Program (LP).

The graphical method is applicable to solve the LPP involving two decision variables x_1 , and x_2 , we usually take these decision variables as x, y instead of x_1, x_2 . To solve an LP, the graphical method includes two major steps.

Linear Programming Problem involving two variables can be solved by Graphical method. Since the solution has to satisfy the non negative conditions the value of the variable x_1 and x_2 can lie only in the first quadrant. The main steps for solving a LPP by graphic method are

1. Formulate the problem into a Linear Programming Problem
2. Each inequality in the constraints may be written as equality
3. Draw straight lines corresponding to the equations obtained in step 2. So there will be many straight line as the constraints are.
4. Identify the feasible region. Feasible region is the area which satisfies all the constraints simultaneously
5. The vertices of the feasible solutions are to be located and their co ordinates are to be measured.
6. Calculate the value of the objective function at each vertex.
7. The solution is the co-ordinate of the vertex, which optimizes the objective function, and the corresponding value of the objective function is the optimum value.

Example

Solve the following LPP

$$\text{Maximize } Z = 3x_1 + 5x_2$$

Subject to

$$2x_1 + 6x_2 \leq 36$$

$$3x_1 + 2x_2 \leq 24$$

$$x_1, x_2, \geq 0.$$

Change the constraints to equality we have

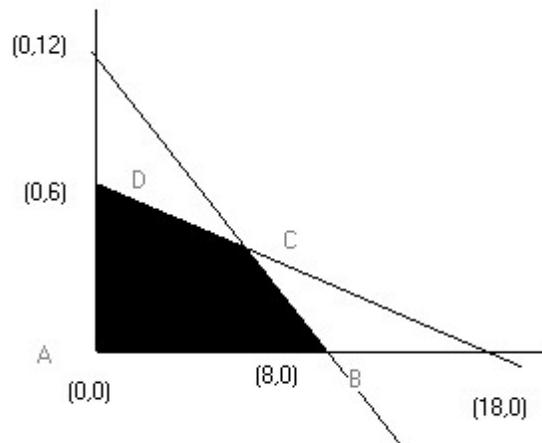
$$2x_1 + 6x_2 = 36 \quad (1)$$

$$3x_1 + 2x_2 = 24 \quad (2)$$

Now consider equation (1), when $x_1 = 0$, $x_2 = 6$ and when $x_2 = 0$, $x_1 = 18$, so the two points in line (1) are $(0,6)$ and $(18,0)$. Similarly for equation (2) we have two points $(0,12)$ and $(8,0)$. Plotting this we get the straight line graphs as

Vertex	Function value of Z
A=(0,0)	0
B=(8,0)	24
C=(36/7,60/7)	58.2857
D=(0,6)	30

So the optimum solution is $(36/7,60/7)$ and the optimum value is 58.2857.



Example-2

$$\text{Max } Z = 50x + 18y$$

Subjected to

$$2x + y \leq 100$$

$$x + y \leq 80$$

$$x, y \geq 0$$

To determine two points on the straight line $2x + y = 100$

Put $y = 0$, $2x = 100 \Rightarrow (50, 0)$ is a point on the line

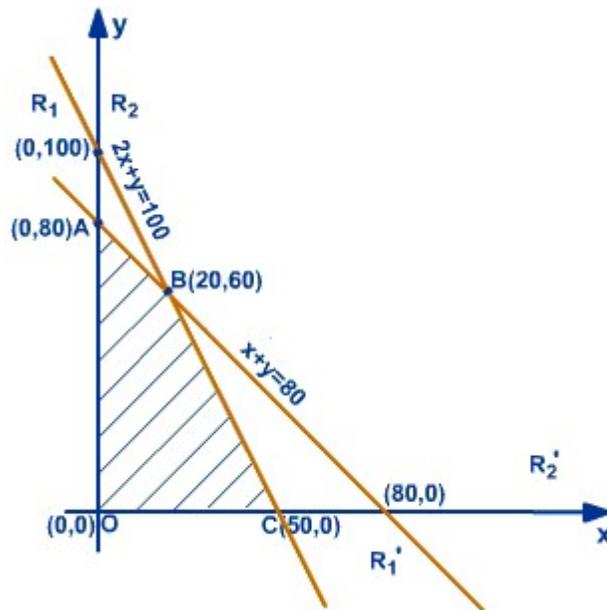
put $x = 0$ in (2), $y = 100 \Rightarrow (0, 100)$ is a point on the line

To determine two points on the straight line $x + y = 80$

Put $y = 0$, $\Rightarrow (80, 0)$ is a point on the line

put $x = 0 \Rightarrow (0, 80)$ is a point on the line

Plotting these lines on the graph paper we have



The intersection of both the region R_1 and R_1' is the feasible solution of the LPP. Therefore every point in the shaded region $OABC$ is a feasible solution of the LPP, since this point satisfies all the constraints including the non-negative constraints.

Vertex	Function value of Z
$O(0,0)$	0
$A(0,80)$	1440
$B(20,60)$	2080
$C(50,0)$	2500

So the optimum solution is $(50,0)$ and the optimum value is 2500

Example-3

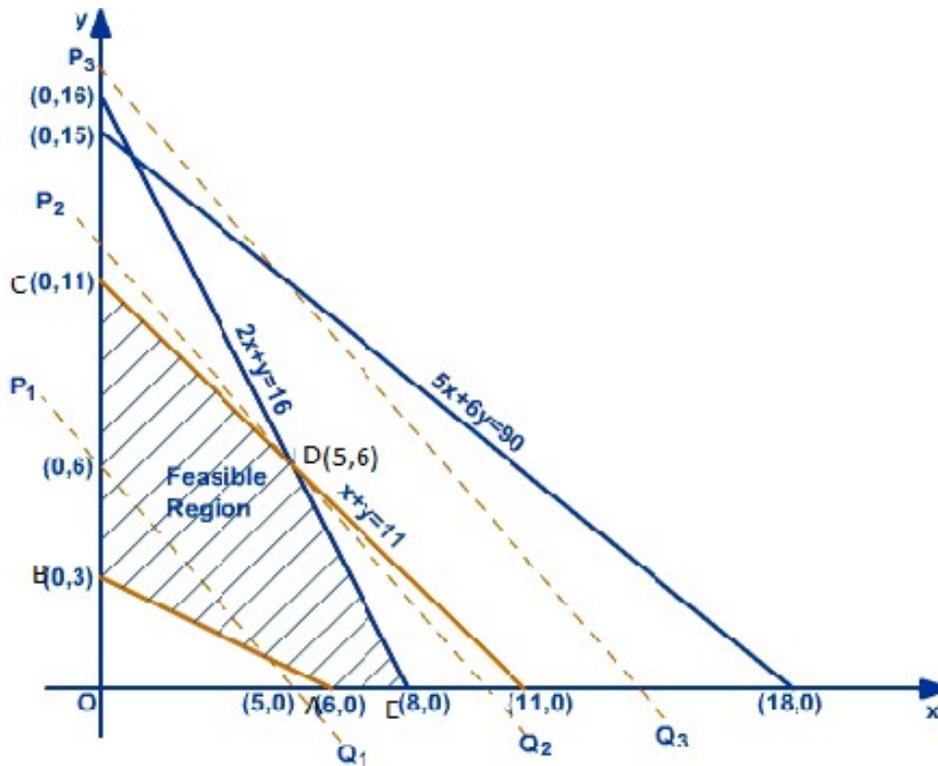
Minimize $Z = 100x + 100y$.
 $10x + 5y \leq 80$
 $6x + 6y \leq 66$
 $4x + 8y \geq 24$
 $5x + 6y \leq 90$
 $x, y \geq 0$

since $x \geq 0, y \geq 0$, consider only the first quadrant of the plane graph the following straight lines on a graph paper

$10x + 5y = 80$ or $2x + y = 16$
 $6x + 6y = 66$ or $x + y = 11$
 $4x + 8y = 24$ or $x + 2y = 6$
 $5x + 6y = 90$

Vertex	Function value of Z
$A(0,6)$	600
$B(0,3)$	300
$C(0,11)$	1100
$D(5,6)$	1100
$E(8,0)$	800

So the optimum solution is $(0,300)$ and the optimum value is 300



SIMPLEX METHOD

Simplex method was originally developed by G.B.Dantzig, an American mathematician.

Simplex method is a Linear Programming technique in which we start with a certain solution which is feasible. We improve this solution in a number of consecutive stages until we arrive at an optimal solution.

For arriving at the solution of LPP by this method, the constraints and the objective function are presented in a table known as simplex table. Then following a set procedure and rules, the optimal solution is obtained making step by step improvement.

Thus Simplex method is an iterative (step by step) procedure in which we proceed in a systematic steps from an initial Basic Feasible Solution to another Basic Feasible Solution and finally, in a finite number of steps to an optimal basic feasible solution, in such a way that the value of the objective function at each step is better (or at least not worse) than that at the preceding steps. In other words the simplex algorithm consists of the following main steps

- (1) Find a trial Basic Feasible Solution of the Linear Programming Problem.
- (2) Test whether it is an optimal solution or not.
- (3) If not optimal, improve the first trial Basic Feasible Solution by a set of rules. That is determine which nonbasic variable should become a basic variable and which basic variable should become a nonbasic variable to find a new bfs with a better objective function value
- (4) Repeat the steps (2) and (3) till an optimal solution is obtained.

How to construct a simplex table?

Simplex table consists of rows and columns. If there are 'm' original variables and 'n' introduced variables, then there will be $3+m+n$ columns in the simplex table. (Introduced variables are slack, surplus or artificial variables).

First column (B) contains the basic variables. Second column (C) shows the coefficient of the basic variables in the objective function. Third column (x_B) gives the values of basic variables. Each of next 'm+n' columns contain coefficient of variables in the constraints, when they are converted into equations.

Basic (B)

The variables whose values are not restricted to zero in the current basic solution, are listed in one column of the simplex table known as Basis (B).

Basic variables

The variables which are listed in the basis are called basic variables and others are known as non-basic variables.

Vector

Any column or row of a simplex table is called a vector. So we have X_1 – vector, X_2 – vector etc.

In a simplex table, there is a vector associated with every variable. The vectors associated with the basic variables are unit vectors.

Unit vector

A vector with one element 1 and all other elements zero, is a unit vector.

Eg: $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ are unit vectors.

Net Evaluation (Δ_j)

Δ_j is the net profit or loss if one unit of the variable in the respective column is introduced. That is, Δ_j shows what is the profit (or loss) if one unit of x_j is introduced. The row containing Δ_j values is called net evaluation row or index row.

$$\Delta_j = C_j - Z_j$$

where c_j is the coefficient of x_j variables in the objective function and z_j is the sum of the products of coefficients of basic variables in the objective function and the vector x_j .

Minimum Ratio

Minimum ratio is the lowest non negative ratio in the replacing ratio column.

The replacing ratio column (θ) contains values obtained by dividing each element in x_B column (the column showing the values of the basic variable) by the corresponding elements in the incoming vector.

Key Column (incoming vector)

The column which has highest negative Δ_j in a maximization problem or the highest positive Δ_j in a minimization problem, is called incoming vector.

Key row (outgoing vector)

The row which relates to the minimum ratio, is the outgoing vector.

Key element

Key element is that element of the simplex table which lies both in the key row and key column.

Iteration

Iteration means step by step process followed in simplex method to move from one basic feasible solution to another.

Computational procedure of simplex method (Simplex Algorithm)

Step1: Formulate the problem into a LPP

Step2: Convert the constraints into equations by introducing the non-negative slack variables or surplus variables wherever necessary.

Step3: Construct starting simplex table.

Step4: Conduct the test of optimality.

This is done by computing net evaluation $\Delta_j = C_j - Z_j$

The solution under test is not optimal if at least one Δ_j is positive for maximization case. If at least one Δ_j is negative, the solution is not optimal for minimization case. Otherwise the solution is optimal.

If solution under test is not optimal, we must proceed to the next step.

Step5: Find incoming and outgoing vectors.

The incoming vector corresponds to highest Δ_j for maximization cases and highest positive Δ_j for minimization cases. Outgoing vector corresponds to minimum ratio.

Step6: The element which is at the intersection of minimum ratio arrow (\leftarrow) and incoming vector arrow (\uparrow) is called the key element. We mark this element in

There should be 1 at the position of key element. If it is not 1 then divide all the elements of the row, containing key element, by the key element. Then add appropriate multiples of the corresponding elements of this changed row to the elements of all other rows. Now obtain the next simplex table with the changes. Improved basic feasible solution can be readout from the simplex table. The solution is obtained by reading B – column and x_B column together.

Step7: Now test the above improved B. F. S. for optimality as in step 4.

If this solution is not optimal then repeat steps (5) and (6) until an optimal solution is finally obtained.

Artificial variable

Artificial variables are fictitious variables. They are incorporated only for computational purposes. They have no physical meaning. Artificial variables are introduced when the constraints are of the type \geq or $=$.

Big – M Method

If an LP has any \geq or $=$ constraints, a starting bfs may not be readily apparent. When a bfs is not readily apparent, the Big M method or the two-phase simplex method may be used to solve the problem.

Big M method is a modified simplex method for solving a LPP when a high penalty cost (or profit) M has been assigned to the artificial variable in the objective function.

When artificial variables are introduced, we include these artificial variables in the basis (B) first. These artificial variables are driven out in the first iteration. For this purpose, we assign a very large M to each artificial variable as coefficient in the objective function. The quantity ' M ' is known as penalty. In maximization cases $-M$ and in minimization cases $+M$ are assigned to the artificial variables as their coefficients in objective functions.

Big M method can be applied to minimization problems as well as maximization problems.

Simplex Method with 'greater-than-equal-to' (\geq) and equality ($=$) constraints

The LP problem, with 'greater-than-equal-to' (\geq) and equality ($=$) constraints, is transformed to its standard form in the following way.

1. One 'artificial variable' is added to each of the 'greater-than-equal-to' (\geq) and equality ($=$) constraints to ensure an initial basic feasible solution.
2. Artificial variables are 'penalized' in the objective function by introducing a large negative (positive) coefficient M for maximization (minimization) problem.
3. Cost coefficients, which are supposed to be placed in the Z-row in the initial simplex tableau, are transformed by 'pivotal operation' considering the column of artificial variable as 'pivotal column' and the row of the artificial variable as 'pivotal row'.
4. If there are more than one artificial variable, step 3 is repeated for all the artificial variables one by one.

Steps

1. Modify the constraints so that the RHS of each constraint is nonnegative (This requires that each constraint with a negative RHS be multiplied by -1 . Remember that if you multiply an inequality by any negative number, the direction of the inequality is reversed!). After modification, identify each constraint as a \leq , \geq or $=$ constraint.
2. Convert each inequality constraint to standard form (If constraint i is a \leq constraint, we add a slack variable s_i ; and if constraint i is a \geq constraint, we subtract an excess variable e_i).
3. Add an artificial variable a_i to the constraints identified as \geq or $=$ constraints at the end of Step 1. Also add the sign restriction $a_i \geq 0$.
4. Let M denote a very large positive number. If the LP is a min problem, add (for each artificial variable) Ma_i to the objective function. If the LP is a max problem, add (for each artificial variable) $-Ma_i$ to the objective function.
5. Since each artificial variable will be in the starting basis, all artificial variables must be eliminated from row 0 before beginning the simplex. Now solve the transformed problem by the simplex (In choosing the entering variable, remember that M is a very large positive number!).

If all artificial variables are equal to zero in the optimal solution, we have found the **optimal solution** to the original problem. If any artificial variables are positive in the optimal solution, the original problem is **infeasible!!!**

Problem 1

Solve the LPP using Simplex Method

$$\text{Maximize } Z = 7x_1 + 5x_2$$

subject to

$$x_1 + 2x_2 \leq 6 \quad \text{--- (1)}$$

$$4x_1 + 3x_2 \leq 12 \quad \text{--- (2)}$$

$$x_1, x_2 \geq 0$$

Step I Convert the inequality constraints in the problem to equality constraint by adding the slack variables

$$x_1 + 2x_2 + s_1 + 0s_2 = 6$$

$$4x_1 + 3x_2 + 0s_1 + s_2 = 12$$

$$x_1 \quad x_2 \quad s_1 \quad s_2 \quad x_B$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 4 & 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

Step II rewrite the problem into new problem with equality constraints

$$\text{Max: } Z = 7x_1 + 5x_2 + 0s_1 + 0s_2$$

subjected to

$$x_1 + 2x_2 + s_1 + 0s_2 = 6$$

$$4x_1 + 3x_2 + 0s_1 + s_2 = 12$$

$$x_1 \geq 0 \quad x_2 \geq 0$$

ie c_j values are 7, 5, 0, 0

Step 3 Construct simplex table.

Simplex Table I

B	C_B	x_B	x_1	x_2	s_1	s_2	$\theta = \frac{x_B}{x_j}$
s_1	0	6	1	2	1	0	$\frac{6}{1} = 6$
s_2	0	12	4	3	0	1	$\frac{12}{4} = 3 \leftarrow \text{Minimum } \theta$
	Z_j		0	0	0	0	
	C_j		7	5	0	0	
	$\Delta_j = Z_j - C_j$		-7	-5	0	0	

↑
Minimum value of Δ_j

B	C_B	x_B	x_1	x_2	s_1	s_2
s_1	0					
x_1	7	$\frac{12}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	0	$\frac{1}{4}$
		(3)	(1)	(3/4)	(0)	(1/4)

Simplex Table II

B	C_B	x_B	x_1	x_2	s_1	s_2	θ
s_1	0	3	0	$\frac{5}{4}$	1	$-\frac{1}{4}$	
x_1	7	3	1	$\frac{3}{4}$	0	$\frac{1}{4}$	
	Z_j		7	$\frac{21}{4}$	0	$\frac{7}{4}$	
	C_j		7	5	0	0	
	$\Delta_j = Z_j - C_j$		0	$\frac{1}{4}$	0	$\frac{7}{4}$	

All $\Delta_j \geq 0$ So optimality is achieved.

So the solution is $x_1 = 3, x_2 = 0$

Optimum $Z = 3 \times 7 + 5 \times 0 = \underline{\underline{21}}$

Maximize $Z = 5x_1 + 3x_2$

subject to $x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$

$x_1, x_2 \geq 0$

Rewriting the problem using slack variable

Maximize $Z = 5x_1 + 3x_2 + 0s_1 + 0s_2 + 0s_3$

subjected to

$x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 2$

$5x_1 + 2x_2 + 0s_1 + s_2 + 0s_3 = 10$

$3x_1 + 8x_2 + 0s_1 + 0s_2 + s_3 = 12$

$x_1, x_2, x_3 \geq 0$

Simplex Table I

B	C_B	x_B	x_1	x_2	s_1	s_2	s_3	$\theta = \frac{x_B}{x_1}$
s_1	0	2	1	1	1	0	0	2 ←
s_2	0	10	5	2	0	1	0	2
s_3	0	12	3	8	0	0	1	4
Z_j			0	0	0	0	0	
C_j			5	3	0	0	0	
$\Delta_j = Z_j - C_j$			-5	-3	0	0	0	

↑

Calculation

	B	C _B	x _B	x ₁	x ₂	s ₁	s ₂	s ₃
	x ₁	5	2	1	1	1	0	0
2 nd ⇒	s ₂	0	0	0	-3	-5	1	0
	s ₃	0		←—————→				

~~2nd row Table 1~~

Second row = second row of 1st table - 5x ~~first row of first~~ first row of this table.

5x row 1 ⇒ 10 5 5 5 0 0

	B	C _B	x _B	x ₁	x ₂	s ₁	s ₂	s ₃
	x ₁	5	2	1	1	1	0	0
	s ₂	0	0	0	-3	-5	1	0
3 rd ⇒	s ₃	0	6	0	5	-3	0	1

3rd row = 3rd row of 1st table - 3x first row of this table.

6 3 3 3 0 0

First Table

	B	C _B	x _B	x ₁	x ₂	s ₁	s ₂	s ₃
	s ₁	0	2	1	1	1	0	0
	s ₂	0	10	5	2	0	1	0
	s ₃	0	12	3	8	0	0	1

Simplex table 2

B	C_B	x_B	x_1	x_2	s_1	s_2	s_3
$2x_1$	5	2	1	1	1	0	0
s_2	0	0	0	-3	-5	1	0
s_3	0	6	0	5	-3	0	1
Z_j			5	5	5	0	0
			5	3	0	0	0
$\Delta_j = Z_j - C_j$			0	2	0	0	0

Since all $\Delta_j \geq 0$ optimum solution is obtained
 Ans $x_1 = 2$ $x_2 = 0$ Max $Z = 5 \times 2 + 3 \times 0 = 10$

☛ Solve the following LPP using simplex Method

1) Max $Z = 40x_1 + 8x_2$

subjected to

$$2x_1 + 3x_2 \leq 48$$

$$x_1 \leq 15$$

$$x_2 \leq 10$$

$$x_1 \geq 0, x_2 \geq 0$$

Ans $x_1 = 9, x_2 = 10$

$$Z = 1160$$

2) Max $Z = 6x_1 + 4x_2$

s.t. $-2x_1 + x_2 \leq 2$

$$x_1 - x_2 \leq 2$$

$$3x_1 + 2x_2 \leq 9$$

$$x_1, x_2 \geq 0$$

Ans $x_1 = \frac{13}{5}, x_2 = \frac{3}{5}$

$$Z = 18$$

3) Max $Z = 3x_1 + 4x_2$

s.t. $2x_1 + 5x_2 \leq 120$

$$4x_1 + 2x_2 \leq 80$$

Ans $x_1 = 10, x_2 = 20$

$$Z = 110$$

Method - III

Solve Max $Z = x_1 + x_2 + x_3$

s.t $4x_1 + 5x_2 + 3x_3 \leq 15$

$10x_1 + 7x_2 + x_3 \leq 12$

Simplex Form

Max $Z = x_1 + x_2 + x_3 + 0s_1 + 0s_2$

$4x_1 + 5x_2 + 3x_3 + s_1 + 0s_2 = 15$ — (1)

$10x_1 + 7x_2 + x_3 + 0s_1 + s_2 = 12$ — (2)

First find initial BFS For this we put

non BFS variable zero i.e. we put $x_1 = 0, x_2 = 0, x_3 = 0$
in (1) and (2) so we get

$s_1 = 15 \quad s_2 = 12$

Starting Simplex Table

B	c_B	x_B	x_1	x_2	x_3	s_1	s_2	$\theta = \frac{x_B}{x_1}$
s_1	0	15	4	5	3	1	0	$\frac{15}{4} = 3.75$
s_2	0	12	10	7	1	0	1	$\frac{12}{10} = 1.2 \leftarrow$
	Z_j		0	0	0	0	0	
	C_j		1	1	1	0	0	
	$\Delta_j = C_j - Z_j$		+1	+1	+1	0	0	

↑

Simplex Table II

B	c_B	x_B	x_1	x_2	x_3	s_1	s_2
s_1	0	$5/5$	0	$43/10$	$29/10$	1	$-4/10$
x_1	1	$6/5$	1	$7/10$	$1/10$	0	$1/10$
	Z_j		1	$7/10$	$1/10$	0	$1/10$
	C_j		1	1	1	0	0
	$\Delta_j = C_j - Z_j$		0	$3/10$	$9/10$	0	$-1/10$

↑

Simplex Table IV

B	c_B	x_B	x_1	x_2	x_3	s_1	s_2	s_3
x_2	5	$\frac{50}{41}$	0	1	0	$\frac{15}{41}$	$\frac{8}{41}$	$-\frac{10}{41}$
x_3	4	$\frac{62}{41}$	0	0	1	$-\frac{6}{41}$	$\frac{5}{41}$	$\frac{4}{41}$
x_1	3	$\frac{89}{41}$	1	0	0	$-\frac{2}{41}$	$-\frac{12}{41}$	$\frac{15}{41}$
Z_j			3	5	4	$\frac{45}{41}$	$\frac{24}{41}$	$\frac{11}{41}$
c_j			3	5	4	0	0	0
$\Delta_j = c_j - z_j$			0	0	0	$-\frac{45}{41}$	$-\frac{24}{41}$	$-\frac{11}{41}$

Since all $\Delta_j \leq 0$ optimality is attained

~~opt~~ optimal solutions are

$$x_1 = \frac{89}{41}, \quad x_2 = \frac{50}{41}, \quad x_3 = \frac{62}{41}$$

$$\text{Max } Z = \frac{765}{41}$$

Solve the following LPP. using Simplex method

① Max $Z = 3x_1 + 2x_2 + 5x_3$

s.t $x_1 + 4x_2 \leq 420$

$3x_1 + 2x_3 \leq 460$

$x_1 + 2x_2 + x_3 \leq 430$

$x_1, x_2, x_3 \geq 0$

② Max $Z = x_1 - x_2 + 3x_3$

$x_1 + x_2 + x_3 \leq 10$

$2x_1 - x_3 \leq 2$

$2x_1 - 2x_2 + 3x_3 \leq 0$

$x_1, x_2, x_3 \geq 0$

③ Max $Z = x_1 + x_2 + x_3$

s.t $4x_1 + 5x_2 + 3x_3 \leq 15$

$10x_1 + 7x_2 + x_3 \leq 12$

$x_1, x_2, x_3 \geq 0$

④ Max $Z = 4x_1 + 3x_2 + 3x_3 + 6x_4$

s.t $x_1 + 2x_2 + 2x_3 + 4x_4 \leq 80$

$2x_1 + 2x_3 + x_4 \leq 60$

$3x_1 + 3x_2 + x_3 + x_4 \leq 80$

$x_1, x_2, x_3, x_4 \geq 0$

Big M method

$$\text{Min } Z = 5x_1 + 6x_2$$

$$\text{s.t. } 2x_1 + 5x_2 \geq 1500$$

$$3x_1 + x_2 \geq 1200$$

$$x_1 \geq 0, x_2 \geq 0$$

$$\text{Min } Z = 5x_1 + 6x_2 + 0s_1 + 0s_2 + MA_1 + MA_2$$

s.t

$$2x_1 + 5x_2 - s_1 + A_1 = 1500$$

$$3x_1 + x_2 - s_2 + A_2 = 1200$$

Converting this problem to Maximization problem

$$\text{Max } Z' = -5x_1 - 6x_2 - 0s_1 - 0s_2 - MA_1 - MA_2$$

s.t

$$2x_1 + 5x_2 - s_1 + 0s_2 + A_1 + 0A_2 = 1500$$

$$3x_1 + x_2 + 0s_1 - s_2 + 0A_1 + A_2 = 1200$$

Starting Simplex Table

B	C_B	X_B	x_1	x_2	s_1	s_2	A_1	A_2	$\theta = X_B/x_1$
A_1	-M	1500	2	5	-1	0	1	0	$\frac{1500}{5} = 300 \leftarrow$
A_2	-M	1200	3	1	0	-1	0	1	$\frac{1200}{1} = 1200$
	Z_j		-5M	-6M	M	M	-M	-M	
	C_j		-5	-6	0	0	-M	-M	
	$A_j = C_j - Z_j$		$5M - 5$	$6M - 6$	-M	-M	0	0	



Simplex Table II

B	C_B	X_B	x_1	x_2	s_1	s_2	A_2	$\theta = \frac{X_B}{A_2}$
x_2	-6	300	$\frac{2}{3}$	1	$-\frac{1}{5}$	0	0	750
A_2	-M	900	$\frac{13}{5}$	0	$\frac{1}{5}$	-1	1	$\frac{4500}{13}$ ←
	Z_j		$-\frac{12}{5} - \frac{13M}{5}$	-6	$-\frac{6}{5} - \frac{M}{5}$	M	-M	
	C_j		5	6	0	0	-M	
	$\Delta_j = C_j - Z_j$		$\frac{13M}{5} + \frac{13}{5}$	0	$\frac{6}{5} - \frac{M}{5}$	-M	0	

↑

Simplex Table III

B	C_B	X_B	x_1	x_2	s_1	s_2
x_2	-6	$\frac{2100}{13}$	0	1	$-\frac{3}{13}$	$\frac{2}{13}$
x_1	-5	$\frac{4500}{13}$	1	0	$\frac{1}{13}$	$-\frac{5}{13}$
	Z_j		-5	-6	1	1
	C_j		5	-6	0	0
	$\Delta_j = C_j - Z_j$		0	0	-1	-1

Since all $\Delta_j \leq 0$ optimality is achieved

$$\text{So. } x_1 = \frac{4500}{13} \quad x_2 = \frac{2100}{13}$$

$$\text{Max } Z = \underline{\underline{2700}}$$

Solve

$$\min Z = 20x_1 + 24x_2 + 18x_3$$

$$\text{s.t. } 2x_1 + x_2 + x_3 \geq 30$$

$$x_1 + x_2 + x_3 \geq 20$$

$$x_1 + 2x_2 + x_3 \geq 24$$

$$x_1, x_2, x_3 \geq 0$$