FUNCTION

The word function describes the manner in which one variable changes in relation to the changes in the other related variables. A function is therefore, a relation which associated any given variable with other variables. It is a rule or a formula that give the value of one variable if the value of the other variable is specified.

Consider the variable x and y such that $y=x^2$. Here every value of y is obtained by squaring the corresponding value of x. So we can say 'y' is a function of x. i.e., y= f(x). Here f describes a rule to get the value of y from any value of x.

For example: Demand and price of a commodity are two variables. They are so related that for a given value of price, we can find a corresponding value of demand. Therefore demand is a function of price.

ALGEBRAIC FUNCTIONS

When the relation between the dependent and independent variables involves only a finite number of terms, and the variables are affected only by the operations of addition, subtraction, multiplication, division and powers, the function is said to be an algebraic function.Eg: $y= 3x^3-5x^2+7x$ is an algebraic function.

Some Algebraic functions

1. **Polynomial function**

An algebraic function is said to be polynomial if it is of the form $y=a_0x^n + a_1x^{n-1}+ \ldots\ldots+a_n$ where a_0 , a_1, ...a_n are constants and n is a positive integer. Therefore in a polynomial function, the powersof independent 'x' in all the terms are positive integers.

Eg: $y= 3x^3-5x^2+7x+6$ is a polynomial of degree3.

2. **Linear function**

A polynomial of degree 1 in x is called a linear function of x. For example, y= 3x+ 2 is a linear function of x.

3. **Quadratic function**
   A polynomial of degree 2 in x is called a quadratic function. For example, $y= 3x^2+ 2x+7$ is called a quadratic function.

4. **System of Linear equations**

A linear equation is an equation in which the unknowns have power 1. A system of linear equations is a set of simultaneous equations with degree 1 for the unknowns.
Eg: $2x+3y=5$, $4x+7y=25$
The value of x and y obtained by solving these two equations will satisfy both the equations.

5. **Functions of one independent variable**

If x and y are two variables such that x is a independent variable and y depends on x then we say y is a function of x denoted by $y = f(x)$.
Eg: $y = 4x^2+5x-2$ is a function of one independent variable 'x'.

6. **Functions of two or more independent variables**

If x, y, and z are three variables such that x and y are two independent variables and z depends on x and y then we say that z is a function of x and y is denoted by $z = f(x,y)$. Eg: $z= 4x^2+5y^2+2xy$ is a function of two variables x and y.

(1) When demand depends on the price, demand is a function of single variable namely price. So we can write $d = 4p-2$

(2) When yield depends on fertilizer and rainfall, yield is a function of two variables fertilizer and rainfall.

7. **Explicit and implicit functions.**

When the dependent variable (y) is expressed directly in terms of the independent variable (x) the relation between x and y is explicitly defined. In an explicit relation a particular value of x corresponds to a particular value of y.
Eg: $y = 3x^2+2x+5$ is an equation which expresses the relation between x and y explicitly, since y us shown in terms of x and constants only. When we put $x=2$, $y=21$. So for a particular value of x, we get a particular value of y.

When the relation between the dependent variable (y) and independent variable (x) is not expressed directly, the relation is implicit. That is, y is not exhaustively expressed in terms of x.

Eg: $y = xy + x^2+ y^2=0$ is an implicit function.

8. **Exponential function**

A simple exponential function is defined by the equation $y = ab^x$. Exponential function is a power function.
Eg: $4(3)^x$ is an exponential function

9. **Logarithmic function**

A function of the form $y=\log_3{}^x$ is called a logarithmic function.

Eg: $\log_3^x$ is a logarithmic function

10. **Logarithmic linear function**

Consider the power function $y=ax^b$. This is a non linear function. Here and b are parameters (ie. Constants). This power function can be converted into a linear function by taking the logarithm of both sides

$y=ax^b$, becomes log y = log a+ b log x

Take log y=Y, log a = A, log x= X. The equation becomes Y=A+bX. This is a linear function where Y, A and X are the logarithmic values of y, a and x respectively. The function Y=A+bX is called log linear function.

Monotonically increasing and decreasing function

If the value of a function always increases as the value of independent variable increases, the function is known as monotonically increasing function. Ie, f(x) increases as x increases.

For example: f(x)=3x+5, is an increasing function

If the value of a function always decreases, as the value of independent variable increasesk the function is known as monotonically decreasing function

Ie, f(x)decreases as x increases. Eg: f(x)= -3x+5

Inverse function

If y is a function of x, we write y =f(x). That is, y is expressed in terms of x. From this function of x, if we can express x in terms of y which is written as x = g(y) or x = $f^{-1}(y)$, then it is called inverse function.

Eg: y=3x+2 shows y as a function of x. Suppose we write from this relation, $x = \frac{y-2}{3}$, then it is the inverse function. Therefore when f(x)=3x+2, then g(y)= $\frac{y-2}{3}$. f(x) is the function and g(y) is the inverse function

ECONOMIC FUNCIONS

Economics is concerned with measurable quantities such as wage, price, national income etc. Economics also deals with the relations among two or more of these quantities. Such relations provide us economic functions. Some important economic functions are

1. **Supply function**

Supply function is a function specifying the quantity of a particular commodity that seller has to offer at various prices. As the price changes supply also changes. That is, supply depends on price. Therefore supply is a function of price. Supply function may be linear or nonlinear.

For example x=3-4p is a linear supply function where x is the quantity of supply and p is the price. But x = $\sqrt{p-4}$ is a non-linear function.

2. **Demand function**
   Demand function is a function specifying the quantity of a particular commodity that buyers are willing to purchase at various prices. As the price changes demand also changes. That is, demand depends on price. Therefore, demand is a function of price. Demand function may be linear or non-linear. For eg, x=60+3p is a linear demand function where x stands for quantity demanded and p for the price. x=$\frac{300}{p+2}$ is a non-linear function.
3. **Cost function**
   Total cost depends on the quantity of output produced. So the total cost is dependent on the quantity of production. That total cost is a function of the quantity of production. For eg, c=4x + 10 is a cost function where x is the quantity of production.

   Cost comprises of fixed cost and variable cost. Fixed cost is denoted by F(x) and variable cost is denoted by V(x). x stands for the quantity of production. F(x) is independent of x. But V(x) depends in x,

   Therefore, Cost function, TC=F(x) + V(x)
4. **Average cost**
   Average cost of production or cost per unit is the ratio between total cost and units of production. The total cost increases as the quantity of production increases. But the average cost decreases as output increases upt a level.
   AC=$\frac{Total\ cost}{x}$
   Eg, Let total cost, TC = $3x^2$+2x+1, then Average cost AC = 3x+2+$\frac{1}{2}$

5. **Revenue function**

Total revenue is the amount of money derived from the sale of a particular commodity. Total Revenue therefore depends on the price of the commodity sold. Total Revenue = x *p where x is the quantity sold and p is the price per unit. Thus total Revenue is a function of price or the quantity sold.

Example of Revenue function (i)TR=4p where p is the price charged and 4 is the quantity sold. (ii) TR=3x where x is the quantity sold and 3 is the price per unit.

6. **Profit function**

Profit is the excess of revenue over the cost of production. That is, Profit=Revenue – Cost=TR-TC. Negative profit indicates loss. Profit function depends on the quantity produced and sold.

Eg: If TR=10x and TC=5x+2. Then Profit function = TR-TC=10x-(5x+2) = 5x-2

7. **Utility function**

Utility is the satisfaction derived by an individual when he purchases particular quantity of a commodity for a particular cost. Therefore utility function depends on the quantity of commodity purchased.

Eg: If a and b are the quantities of commodities X and Y purchased then the utility is a function of a and b. So U=5a+2b is an example of a utility function.

8. **Consumption function**

Consumption function depends on disposable income. Let 'C' be the consumption and y be the disposable income. Then C is a function of y. That is, C=f(y)

$C=\frac{3}{4}y$ is an example of consumption function.

9. **Production function**

There is a functional relation between inputs and outputs which is usually referred to as the production function. Input refers to the factors like labour, capital, land, etc. Output refers to the goods produced.

For eg: (1) $f(L,K)=4+L^2+K^2-Lk$ is a production function. (2) $q=aL^pk^q$ is another example of production function known as Cobb-Douglar function.

10. **Saving function**

Saving function may be defined as the difference between income and consumption. Saving is positive or negative according as the income exceeds consumption or consumption exceeds income.

S=y-c where y is the income and c is the consumption since c is a function of y, 's'=y-c is also a function of y.

For eg: s=a+by is a saving function, where a and b are constants

11. **Investment functions**

Investment depends on income. Therefore investment is a function of income. For example, I=y-5 is an investment function where y is the income.

PRODUCTION FUNCTION

The theory of production plays two important roles in the theory of relative prices. It provides a base for the analysis of relations between costs and volumes of input costs influence supplies which along with demand determine prices. The theory of production serves a base for the theory of demand for factors of production. Production means a transformation of inputs into outputs. The inputs are the things brought by the firm and the outputs are the things which the firm sells.

Production takes place by the combined forces of various factors of production such as land, labour capital and enterprise. In production there are two processes, namely theory of production and theory of costs. The theory of production shows the physical relationship between inputs and outputs. The theory of costs represent the level of output and costs. A production function is a technique which represents the technology involved in the process of production.

Production explains the relationship between the inputs and outputs. The output of a commodity depends on the employed inputs. Production function explains the physical relationship between the units of employed inputs and the output. The money price do not appear in a production function.

Mathematically X = f(L,K,l,E)
When X = output, L =labour, K= capital, l = land and E = Enterprise

The output in the dependent variable and all others are independent variables. If any firm wants to increase its production it can do in two ways. Either it can increase all the units of required inputs or can increase the units of some inputs white others remain constant.

Production not only depends on the quantity of inputs but also depend upon the technique of production. The improvement in technology results in increased production, without any increase in inputs. In this case production function itself changes. The production function depends upon two things i) The quantity of production and ii) The range of production techniques. The possibility of substituting different factors of production play an important role in the production process.

Homogeneous function

A function $f(x, y)$ is said to be a homogeneous function of order *n* if

$$f(tx, ty) = t^n f(x, y)$$

Example: Let $f(x, y) = x^2 y^4$

Then

$$f(tx, ty) = (tx)^2(ty)^4 = t^6x^2y^4 = t^6f(x, y)$$

So $f(x, y)$ is a homogeneous function of degree 6.

Multivariate functions that are "homogeneous" of some degree are often used in economic theory. A function is homogeneous of degree k if, when each of its arguments is multiplied by any number $t > 0$, the value of the function is multiplied by t^k.

Let f is a function on n variables $(x_1, x_2, \dots, x_n)$. Then f is said to be a homogenous function of degree k if, for any $t > 0$,

$$f(tx_1, tx_2, \dots, tx_n). = t^k f(x_1, x_2, \dots, x_n).$$

Linear Homogeneity

Homogeneity of order one is called linear homogeneity

Linear Homogeneous function

Let f is a function on n variables $(x_1, x_2, \dots, x_n)$. Then f is said to be a linear homogenous function if, for any $t > 0$,

$$f(tx_1, tx_2, \dots, tx_n). = tf(x_1, x_2, \dots, x_n).$$

Cobb – Douglas Production Function

The general form of Cobb – Douglas Production Function is

$$Q = AK^{\alpha}L^{\beta},$$

where Q = output, K = capital, L = labour
A is a constant α and β are positive parameters
K and L are the capital and labour inputs.

If α + β = 1, the production function will operate under constant return to scale.
If α + β > 1, there will be increasing returns to scale.
If α + β < 1, there will be decreasing returns to scale.

Properties

a) If the inputs are increased by t times, the total output will also increase by t times
b) The production function is homogeneous of degree one if α + β = 1
c) If the production function is linear and homogeneous, then elasticity of substitution equal to unity
d) α and β represents the capital share and labour share to the total output respectively.
e) α and β represent the elasticity's of output with respect to capital and labour respectively.

f) The expansion path of Cobb – Douglas production function is linear homogeneous and passes through origin

Importance of Cobb – Douglas production function

Cobb – Douglas production function is most commonly used function in Economics. This function is helpful in wage determination principles. This function is helpful in explaining marginal productivity principles and used to explain product exhaustion theorem. The parameter α and β represent elasticity coefficients. They are helpful in comparing international levels. This function is helpful in the study of different laws of returns to scale. This function plays an important role in economic field. It is used to determine wage policies under sector comparisons, substitutability and the degree of homogeneity.

Limitations:

Cobb – Douglas production function has certain draws backs.

a) If contains only two inputs, capital and labour. There are several other factors which are equally important in production.
b) The production function operates under constant returns to scale. The conditions of increasing returns and diminishing returns are ignored.
c) The function assumes that technological conditions remains constant. But it is not time. The output conditions vary with the change in technological conditions.
d) The function assumes that all inputs are homogeneous. In practice, all factors are not equally efficient.
e) It assumes perfect competition. But if there is imperfect competition this function does not hold good.
f) This function ignores the negative marginal productivity of a factor.

Constant Elasticity Production Function (C.E.S. function)

Cobb – Douglas production function is a function whose elasticity of substitution is unity every where. Any production whose elasticity of substitution is constant but other than unity is called Constant Elasticity substitution production function or C.E.S. Production function.

C.E.S Production function represents the more general form of production technique than Cobb -Douglas production function. In C.E.S. Production function, $\sigma \neq 1$, but a constant C.E.S. Production function has wider scope, substitutability and efficiency.

C.E.S. Production function removes all the difficulties and unrealistic assumptions of Cobb-Douglas production function. Cobb – Douglas production function is a special case of C.E.S. production function.
A.C.E.S. Production function is expressed as

$$Q = [\delta C^{-\alpha} + (1-\delta)N^{-\alpha}]^{\frac{-v}{\alpha}},\ (v>0\ 0<\delta<1 \text{ and } \alpha > -1)$$

Where Q = output, C = Capital input, N = labour input, α = Substitution parameter,
1-δ = labour intensity coefficient and V = degree of freedom

C.E.S. Production function has some limitations

1. It consider only two factor N and C.
2. C.E.S. function contains only one parameter, V, which is affected by the scale of operation and technological change, There two factors may affect the degree of returns to scale but cannot distinguish them separately.
3. It is assumed that in this function that '?'changes in response to technology only and factor proportions do not affect it.
4. While the empirical study shows that the elasticity of substitution also changes due to changed factor proportions. The function has ignored this important fact.

EULER'S THEOREM (Leonhard Euler (1707-1783))

Euler's theorem states that when all factors of production are increased in a given proportion, the resulting output will also increase in a given proportion, provided each factors of production is paid the value of its marginal product and the total output is just exhausted.

Theorem:- If $Q = f(K, L)$ is a linearly homogeneous function, then

$$\boldsymbol{K\frac{\delta Q}{\delta K} + L\frac{\delta Q}{\delta L} = Q.}$$

Consider the Cobb – Douglas Production Function $Q = AK^{\alpha}L^{\beta}$, with α + β = 1. Then

$$\frac{\delta Q}{\delta K} = \boldsymbol{\alpha} AK^{\alpha-1}L^{\beta}.$$

$$\frac{\delta Q}{\delta L} = \boldsymbol{\beta} AK^{\alpha}L^{\beta-1}.$$

$$\boldsymbol{K\frac{\delta Q}{\delta K} + L\frac{\delta Q}{\delta L}} = (\alpha + \beta)\ AK^{\alpha}L^{\beta} = AK^{\alpha}L^{\beta} = \boldsymbol{Q}.$$

Since α + β = 1.

Euler's theorem plays an important part in the economic field especially in the field of distribution. The theorem suggests to a firm how the inputs should be employed. The input should be employed to that extent at which the price of factor is equal to its marginal revenue product. Thus it also suggests the determination of price of a factor of productions. Euler's theorem is based on following assumption.

a) The law of constant returns to scale is being applied. It holds good only in the case of a linearly homogeneous production function of degree one.
b) There is perfect competition in the market .
c) It assumer perfect divisibility of the factors of production.
d) The technology remain constant in the given time period.