PRODUCTION FUNCTION

The theory of production plays two important roles in the theory of relative prices. It provides a base for the analysis of relations between costs and volumes of input costs influence supplies which along with demand determine prices. The theory of production serves a base for the theory of demand for factors of production. Production means a transformation of inputs into outputs. The inputs are the things brought by the firm and the outputs are the things which the firm sells.

Production takes place by the combined forces of various factors of production such as land, labour capital and enterprise. In production there are two processes, namely theory of production and theory of costs. The theory of production shows the physical relationship between inputs and outputs. The theory of costs represent the level of output and costs. A production function is a technique which represents the technology involved in the process of production.

Production explains the relationship between the inputs and outputs. The output of a commodity depends on the employed inputs. Production function explains the physical relationship between the units of employed inputs and the output. The money price do not appear in a production function.

Mathematically X = f(L,K,l,E)
where X = output, L =labour, K= capital, l = land and E = Enterprise

The output is the dependent variable and all others are independent variables. If any firm wants to increase its production it can do in two ways. Either it can increase all the units of required inputs or can increase the units of some inputs white others remain constant.

Production not only depends on the quantity of inputs but also depend upon the technique of production. The improvement in technology results in increased production, without any increase in inputs. In this case production function itself changes. The production function depends upon two things i) The quantity of production and ii) The range of production techniques. The possibility of substituting different factors of production play an important role in the production process.

Homogeneous function

A function $f(x, y)$ is said to be a homogeneous function of order *n* if

$$f(tx, ty) = t^n f(x, y)$$

Example: Let $f(x, y) = x^2 y^4$

Then

$$f(tx, ty) = (tx)^2(ty)^4 = t^6x^2y^4 = t^6f(x, y)$$

So $f(x, y)$ is a homogeneous function of degree 6.

Multivariate functions that are "homogeneous" of some degree are often used in economic theory. A function is homogeneous of degree k if, when each of its arguments is multiplied by any number $t > 0$, the value of the function is multiplied by t^k.

Let f is a function on n variables $(x_1, x_2, \dots, x_n)$. Then f is said to be a homogenous function of degree k if, for any $t > 0$,

$$f(tx_1, tx_2, \dots, tx_n). = t^k f(x_1, x_2, \dots, x_n).$$

Linear Homogeneity

Homogeneity of order one is called linear homogeneity

Linear Homogeneous function

Let f is a function on n variables $(x_1, x_2, \dots, x_n)$. Then f is said to be a linear homogenous function if, for any $t > 0$,

$$f(tx_1, tx_2, \dots, tx_n). = tf(x_1, x_2, \dots, x_n).$$

Cobb – Douglas Production Function

The general form of Cobb – Douglas Production Function is

$$Q = AK^{\alpha}L^{\beta},$$

where Q = output, K = capital, L = labour
A is a constant α and β are positive parameters
K and L are the capital and labour inputs.

If α + β = 1, the production function will operate under constant return to scale.
If α + β > 1, there will be increasing returns to scale.
If α + β < 1, there will be decreasing returns to scale.

Properties

a) If the inputs are increased by t times, the total output will also increase by t times
b) The production function is homogeneous of degree one if α + β = 1
c) If the production function is linear and homogeneous, then elasticity of substitution equal to unity
d) α and β represents the capital share and labour share to the total output respectively.
e) α and β represent the elasticity's of output with respect to capital and labour respectively.

f) The expansion path of Cobb – Douglas production function is linear homogeneous and passes through origin

Importance of Cobb – Douglas production function

Cobb – Douglas production function is most commonly used function in Economics. This function is helpful in wage determination principles. This function is helpful in explaining marginal productivity principles and used to explain product exhaustion theorem. The parameter α and β represent elasticity coefficients. They are helpful in comparing international levels. This function is helpful in the study of different laws of returns to scale. This function plays an important role in economic field. It is used to determine wage policies under sector comparisons, substitutability and the degree of homogeneity.

Limitations:

Cobb – Douglas production function has certain draws backs.

a) If contains only two inputs, capital and labour. There are several other factors which are equally important in production.
b) The production function operates under constant returns to scale. The conditions of increasing returns and diminishing returns are ignored.
c) The function assumes that technological conditions remains constant. But it is not time. The output conditions vary with the change in technological conditions.
d) The function assumes that all inputs are homogeneous. In practice, all factors are not equally efficient.
e) It assumes perfect competition. But if there is imperfect competition this function does not hold good.
f) This function ignores the negative marginal productivity of a factor.

Constant Elasticity Production Function (C.E.S. function)

Cobb – Douglas production function is a function whose elasticity of substitution is unity every where. Any production whose elasticity of substitution is constant but other than unity is called Constant Elasticity substitution production function or C.E.S. Production function.

C.E.S Production function represents the more general form of production technique than Cobb -Douglas production function. In C.E.S. Production function, $\sigma \neq 1$, but a constant C.E.S. Production function has wider scope, substitutability and efficiency.

C.E.S. Production function removes all the difficulties and unrealistic assumptions of Cobb-Douglas production function. Cobb – Douglas production function is a special case of C.E.S. production function.
A.C.E.S. Production function is expressed as

$$Q = [\delta C^{-\alpha} + (1-\delta)N^{-\alpha}]^{\frac{-v}{\alpha}},\ (v>0\ 0<\delta<1 \text{ and } \alpha>-1)$$

Where Q = output, C = Capital input, N = labour input, α = Substitution parameter,
1-δ = labour intensity coefficient and V = degree of freedom

C.E.S. Production function has some limitations

1. It consider only two factor N and C.
2. C.E.S. function contains only one parameter, V, which is affected by the scale of operation and technological change, There two factors may affect the degree of returns to scale but cannot distinguish them separately.
3. It is assumed that in this function that '?'changes in response to technology only and factor proportions do not affect it.
4. While the empirical study shows that the elasticity of substitution also changes due to changed factor proportions. The function has ignored this important fact.

EULER'S THEOREM (Leonhard Euler (1707-1783))

Euler's theorem states that when all factors of production are increased in a given proportion, the resulting output will also increase in a given proportion, provided each factors of production is paid the value of its marginal product and the total output is just exhausted.

Theorem:- If $Q = f(K, L)$ is a linearly homogeneous function, then

$$K\frac{\delta Q}{\delta K} + L\frac{\delta Q}{\delta L} = Q.$$

Consider the Cobb – Douglas Production Function $Q = AK^{\alpha}L^{\beta}$, with α + β = 1. Then

$$\frac{\delta Q}{\delta K} = \alpha AK^{\alpha-1}L^{\beta}.$$

$$\frac{\delta Q}{\delta L} = \beta AK^{\alpha}L^{\beta-1}.$$

$$K\frac{\delta Q}{\delta K} + L\frac{\delta Q}{\delta L} = (\alpha+\beta)\, AK^{\alpha}L^{\beta} = AK^{\alpha}L^{\beta} = Q.$$

Since α + β = 1.

Euler's theorem plays an important part in the economic field especially in the field of distribution. The theorem suggests to a firm how the inputs should be employed. The input should be employed to that extent at which the price of factor is equal to its marginal revenue product. Thus it also suggests the determination of price of a factor of productions.
Euler's theorem is based on following assumption.

a) The law of constant returns to scale is being applied. It holds good only in the case of a linearly homogeneous production function of degree one.
b) There is perfect competition in the market .
c) It assumer perfect divisibility of the factors of production.
d) The technology remain constant in the given time period.